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## THE CONCEPT OF BRIGHTNESS IN CONNECTION WITH BLACKOUT PROBLEMS

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The classical concept of brightness determined with the help of the standard eye-sensitivity curve leads to difficulties in cases where one is concerned simultaneously with low levels of brightness and different coloured light sources (road lighting, blackout problems). The different possibilities to be considered in introducing a more suitable concept of brightness are discussed, and the determination of the quantity concerned is divided into a definition of equality (when may two brightnesses be considered equal?) and a definition of degree (what value must be assigned to a given brightness?). The mathematical formulation only becomes simple when we assume the validity of the summation law for estimating the brightness: the brightness then becomes an arbitrary function of  $S(\lambda) V(\lambda) d\lambda$ , where  $S(\lambda)$  represents the spectral distribution of energy and  $V(\lambda)$  the eye-sensitivity curve under the given conditions of observation. By the choice of different values of this function the definitions of degree of different concepts of brightness are obtained, such as Purkyne brightness, "Dunkelleuchtdichte", etc. The validity or non-validity of the summation law under various conditions of observation is discussed in detail. In conclusion observability of coloured point sources of light is discussed. For the calculation of the threshold value of visibility it is found that it is not permissible to start with the eye-sensitivity curve for rod vision, and that no summation law holds. From this it follows that the so-called "Dunkelbeleuchtungsstärke" which occurs at the eye is not a measure independant of colour for the observability of a light point.

In a previous article published in this periodical<sup>1)</sup> it was explained how it was desirable in the study of the problems of modern road lighting, where it is a question of relatively low levels of brightness and very different colours, to introduce in addition to the customary concept of brightness a new concept which is more closely related with our intuitive concept of "brightness" under the given conditions.

In connection with the problems of blackout where still much lower brightnesses occur, and where the unsuitability of the ordinary idea of brightness is particularly striking, this problem has recently attracted considerable attention.

In the following we shall therefore consider the problem of the determination of the concept of brightness again, and this time from a general point of view. After having explained the requirements which every definition of brightness must satisfy, we shall study more closely the different possibilities of arriving at a practically useful determination.

### The definition of equality

When the eye observes two luminous planes adjacent to each other, it is very difficult to estimate

the brightness relation between the two planes; on the other hand the eye can indeed judge which plane is the brighter, and we can adjust the brightness of one of the planes with considerable accuracy so that the two planes give the impression of being "equally bright".

In setting up a definition of brightness we must keep this property of the eye in mind, so that the first condition which the definition must satisfy is the following.

When two planes give to the eye the impression of being "equally bright" the newly defined brightness must have the same value for both cases<sup>2)</sup>.

This requirement may be considered as the "definition of equality": by definition we have determined under what circumstances we shall consider two brightnesses equal. This definition will of course only have a practical value when by means of an extensive experimental investigation we have ascertained the quantities of light of different colour which must be emitted under various conditions

<sup>1)</sup> Philips techn. Rev. 1, 142, 1936.

<sup>2)</sup> It is clear that this requirement can only be satisfied when there is transitivity of the impressions of brightness, in other words: when two planes *A* and *B* give the impression of being "equally bright" and the same is true of *B* and *C*, then *A* and *C* upon direct comparison also give the impression of being equally bright. This transitive law is found to be valid under all conditions.



to give the average observer the impression of equal brightness. This investigation has by no means been completed for all conditions occurring in practice.

The definition of equality is already sufficient for the treatment of many problems. For certain problems, however, it will be a great advantage to add a "definition of degree" which enables us to assign to a brightness a definite value. The definition of equality answers the question "When are two brightnesses equal?", the definition of degree the question "How great is the brightness?"

### The definition of degree in general

When we consider how to introduce a definition of degree it is immediately clear that we may do this arbitrarily to a certain extent. This fact is connected with the fact that the eye is not able to estimate directly the ratio of the brightnesses of two planes. In the choice of the definition of degree we are, however, bound by two conditions:

- 1) Care must be taken not to come into conflict with the definition of equality.
- 2) When one of two planes is found to be brighter than the other, the brightness of the first must be assigned a higher value.

The most obvious method, and therefore the one appearing earliest in the literature (König 1891), is the following: an arbitrary kind of comparison light is chosen, and by definition it is established that for that kind of light the brightness increases proportionally with the energy radiated per unit of surface. This establishes for this one kind of light a complete scale of brightnesses, while for all other kinds of light the value of the brightness can be determined by means of the definition of equality<sup>3</sup>). It is clear that in this way the two conditions mentioned above are automatically fulfilled.

The kind of comparison light spoken of may be chosen quite arbitrarily, and different formulations of the concept of brightness occurring in the literature are distinguished from each other only in the choice of the comparison light. We shall give a few examples.

König used as comparison light a spectral colour with the wave length 5 350 Å. This colour was also used by the present author in defining the concept of "subjective brightness".

The concept of "Dunkelleuchtdichte" which has been introduced in Germany is determined in quite the same way, but as comparison light the radiation

of a black body with a temperature of 2 360° K<sup>5</sup>) has been chosen. Such a choice has the advantage that with a visual photometer, containing as comparison lamp an electric lamp of this colour temperature, the "Dunkelleuchtdichte" can be correctly measured directly (provided the eye of the observer is normal and well adapted). Since the concept of "Dunkelleuchtdichte" was introduced especially for blackout problems, in working out a method of calculation for the "Dunkelleuchtdichte" the investigation could be limited to those brightnesses at which pure rod vision occurs. For greater brightnesses this concept may not be used.

Quite a different example of a definition of degree would be to choose the brightness scale for a single definite kind of light so that with that kind of light the logarithm of each brightness ratio  $B_1 : B_2$  is a direct measure of the number of steps lying between  $B_1$  and  $B_2$  that have a difference in brightness which is just observable (law of Weber and Fechner). For other kinds of light also this law would then be approximately valid. Over against this, however, are two disadvantages, namely the awkwardness of the concept for the purposes of calculation and the fact the definition does not pass over at high brightnesses into the ordinary brightness definition, as is the case for the other two definitions of degree.

### Eye-sensitivity curve and summation law

In the further development of the definition of brightness the ideas of "eye-sensitivity curve" and "summation law" play an important part. An eye-sensitivity curve is obtained by determining for a given level of brightness the energy (in watts/cm<sup>2</sup> for instance) which must be emitted in the form of different monochromatic radiations to give the eye the impression of "equal brightness". When this energy has the value  $s(\lambda)$  for a given wave length  $\lambda$ , while  $s(\lambda_0)$  is the value for the fixed comparison wave length  $\lambda_0$  for which the energy in question is a minimum, the eye-sensitivity curve is the function  $V(\lambda) = s(\lambda_0) : s(\lambda)$ . For  $\lambda = \lambda_0$ ,  $V(\lambda)$  assumes the value of unity for all other wave lengths  $V(\lambda) < 1$ . The shape of the function  $V(\lambda)$  depends very closely upon the conditions under which the comparison takes place (level of brightness, size of the field of vision, method of photometry, etc.). These conditions must be kept constant during the recording of an eye-sensitivity curve and they must of course also be taken into account in the application of the eye-sensitivity curve.

<sup>3</sup>) In the corresponding acoustic case (the determination of the concept of loudness) the same method is used: as comparison sound a tone is chosen of 1 000 c/s.

<sup>4</sup>) Philips techn. Rev. 1, 142, 1936.

<sup>5</sup>) A. Dresler, Das Licht 10, 112, 118, 145, 1940.



By the summation law we mean the following property of brightness impressions.

When a radiation of energy  $S_1$  (spectral distribution 1) and a radiation of energy  $S_2$  (spectral distribution 2) give the same impression of brightness, this is also true of the mixture  $aS_1 + (1-a)S_2$ , where  $a$  is any arbitrary number between 0 and 1.

#### *The validity of the summation law*

A detailed investigation of the validity of the summation law at low brightnesses and with fairly large fields of vision was carried out by the writer<sup>6)</sup>. The following colours were chosen: the light of incandescent filament lamps, fairly saturated red, blue and green (filtered electric light) and the mixed colours formed from these by mixing two colours in arbitrary proportions. All the colours were compared directly with electric light. For each mixture the brightness to be expected from the summation law was determined and compared with that measured. The range of brightnesses extended from  $5 \times 10^{-5}$  to 0.3 c.p./m<sup>2</sup>, and thus included the greater part of the region important in road lighting and the whole region with which we are concerned in blackout problems. With fields of vision of at least several degrees the summation law was found to hold very accurately in this range of brightnesses. In the comparison between calculated and measured values the average deviations amounted to about 1 per cent and exhibited quite the character of accidental errors. Only at the highest brightnesses used were the measurements more or less non-reproducible in the neighbourhood of the saturated blue, and there were indications that deviations from the summation law began to occur here.

Much work has already been done on the question of the validity of the summation law in the region of high brightnesses (pure cone vision), but without reaching consistent results on this point, which is of extremely great importance in photometry<sup>7)</sup>. The main cause of this lack of agreement is that in this region of brightness, where the colour difference is not weakened by the collaboration of the rods, the "natural" method of measuring the brightness, namely the direct comparison of two different coloured planes side by side, is extremely difficult and leads to very irreproducible results. Some authors (Helmholtz, v. Kries) even doubt whether such measurements have any value at all. For this reason attempts were soon made to replace this unsatisfactory method of photometry by other

methods which must satisfy the requirements of accuracy and reproducibility, of not deviating too much in their results from those of direct comparison and of obeying the summation law. After several vain attempts two methods were found which not only satisfied the requirements made of them, but which also gave good mutual agreement in their results. These were the flicker method<sup>8)</sup> and the step-by-step method<sup>9)</sup>. In 1924 it was therefore decided to standardize a mean of the best of these measurements carried out by both the methods as international eye-sensitivity curve  $V_k(\lambda)$ , and to base the concept of brightness for high levels of brightness upon this curve. Since then the criterion for the excellence of a photometric method is the question of whether or not it produces results which agree with this standardized definition within the limits of accuracy required for the purpose in view.

Although this created a very satisfactory situation for practical work, numerous investigations were still carried out on the mutual agreement between the different photometric methods and on the validity of the summation law. The most important results are the following:

- 1) For the flicker photometer the summation law is very accurately valid (to within 1 to 2 per cent<sup>10)</sup>).
- 2) The step-by-step method gives results which agree within several per cent with those from the flicker photometer<sup>9c)</sup>, and here also the summation law is valid. Although numerous investigators confirmed these two facts, they were pertinently contradicted by others<sup>11)</sup>.
- 3) The method of direct comparison does not lead to reproducible results. According to certain authors<sup>12)</sup> there is good agreement with the flicker photometer, according to others there are fairly large, but unsystematic deviations<sup>8b)</sup>, others again find systematic deviations of 10-20 per cent<sup>8c)</sup>, or even of 50-100 per cent<sup>11, 13)</sup>. The last authors find the greatest deviations for

<sup>8a)</sup> H. E. Ives, *Phil. Mag.*, **24**, 149, 1912.

<sup>b)</sup> W. W. Coblentz and W. B. Emerson, *Bull. Bur. Stand.*, **14**, 167, 1918.

<sup>9a)</sup> H. E. Ives, *Phil. Mag.*, **24**, 744, 1912.

<sup>b)</sup> E. P. Hyde, W. E. Forsythe and F. E. Cady, *Astrophys. J.*, **48**, 87, 1918.

<sup>c)</sup> K. S. Gibson and E. P. T. Tyndall, *Sc. Papers, Bur. Stand.*, **19**, 131, 1923.

<sup>10)</sup> The small deviations (2%) found by Jaggi may probably be ascribed for a large part to the lack of absolute constancy of the eye-sensitivity curve.

<sup>11)</sup> A. Kohlrausch, *Das Licht*, **5**, 259, 275, 1935.

<sup>12)</sup> R. G. Weigel, *Das Licht*, **5**, 43, 1935.

<sup>13)</sup> A. Dresler, *Das Licht*, **7**, 81, 107, 1937.

<sup>6)</sup> *Proc. Kon. Akad. Wet. Amsterdam*, **38**, 150, 1935.

<sup>7)</sup> Cf. for this part also *Philips techn. Rev.*, **1**, 120, 1936 and **5**, 283, 1940.



saturated red, blue-green and blue, the smallest for yellow.

The cause of these discrepancies is not yet completely explained. Many factors which are difficult to control play an important part: technique, and speed of adjustment, the experience of the observer, the psychological attitude, the ability to abstract the colour difference, consciously or unconsciously applied devices of accomplishing this latter, etc.

In the cases in which the above-mentioned large deviations from the results obtained with the flicker photometer were found, the summation law also fails to hold: too high brightnesses are assigned to the saturated colours at the extremities of the spectrum, and this phenomenon disappears when these colours are mixed to give an unsaturated colour. Thus lower values of the brightness are found for the mixture than would be expected according to the summation law.

#### Summation law and definitions of degree

If the validity of the summation law is assumed the following property can be derived from the definition of the eye-sensitivity curve  $V(\lambda)$ .

Two planes which emit a radiation with the spectral distribution  $S_1(\lambda)$  and  $S_2(\lambda)$  will give the impression of being "equally bright" when

$$\int S_1(\lambda) V(\lambda) d\lambda = \int S_2(\lambda) V(\lambda) d\lambda,$$

where  $V(\lambda)$  represents the eye-sensitivity curve that is valid under the conditions at which the comparison takes place.  $S(\lambda) d\lambda$  is the energy which is emitted in the wave-length region between  $\lambda$  and  $\lambda + d\lambda$  per  $\text{cm}^2$  (projected in the direction of observation), per second and per unit of solid angle (steradian).

From the definition of  $s(\lambda)$  as an intensity which stimulates the same impression of brightness for all wave lengths, it follows directly with the help of the summation law that the mixture  $a_1 s(\lambda_1) + a_2 s(\lambda_2)$  gives the same impression of brightness as  $s(\lambda_1)$  or  $s(\lambda_2)$  of  $a_1 + a_2 = 1$ . If this property is extended to more components, and if one then passes over to a continuous spectrum by transition of the limits, it is found that the mixture  $\int a(\lambda) s(\lambda) d\lambda$  makes the same impression of brightness as  $s(\lambda)$  if  $\int a(\lambda) d\lambda = 1$ . If  $a(\lambda) s(\lambda)$  is set equal to  $S(\lambda)$ , it follows that for two spectral distributions  $S_1(\lambda)$  and  $S_2(\lambda)$  which give the same impression of brightness the following is valid:

$$\int \frac{S_1(\lambda)}{s(\lambda)} d\lambda = \int \frac{S_2(\lambda)}{s(\lambda)} d\lambda$$

or, introducing the eye-sensitivity curve  $V(\lambda) = s(\lambda_3)/s(\lambda)$

$$\int S_1(\lambda) V(\lambda) d\lambda = \int S_2(\lambda) V(\lambda) d\lambda.$$

This relation enables us to arrive at a common mathematical formulation of all the possible def-

initions of degree. The new brightness which is to be introduced is connected, according to the above, unambiguously with the expression  $\int S(\lambda) V(\lambda) d\lambda$ , so that we find directly as the most general mathematical formulation of the definition of degree:

$$B = f \int S(\lambda) V(\lambda) d\lambda \{ . . . . (1)$$

In this equation  $f$  represents an arbitrary function of which it is only required that upon continuous growth of the integral,  $B$  shall also increase continuously. Formula (1) is of course only valid in conditions under which the summation law is valid<sup>14</sup>). For great brightnesses this is the case, as stated, only if the flicker photometer is used as a measuring instrument. Since at the same time one also finds very approximately the same eye-sensitivity curve  $V_k(\lambda)$ , the simplest definition of degree for great brightnesses is

$$B_c = C_k \int S(\lambda) V_k(\lambda) d\lambda, . . . . (2)$$

where  $V_k(\lambda)$  is the internationally established standard eye-sensitivity curve and  $C_c$  is a constant which takes on the value 636 when the brightness  $B_c$  is expressed in stilb ( $\text{c.p./cm}^2$ ) and  $S(\lambda)$  in  $\text{watts/cm}^2$ <sup>15</sup>). This formulation is internationally established and the concept of brightness thus defined will in the following be called the "classical brightness" and indicated by the letter  $B_c$ .

When we do not confine ourselves to great brightnesses, the simplest relation between brightness and the integral is also:

$$B = C \int S(\lambda) V(\lambda) d\lambda, . . . . (3)$$

a definition which was proposed by Voet-Mogendorff<sup>16</sup>) among others under the name of the Purkyne brightness. If  $C$  is chosen equal to  $C_c$ , then for the conditions under which the standard eye-sensitivity curve is measured, equation (3) passes over into the classical brightness  $B_c$  according to equation (2).

The definitions with a comparison light discussed

<sup>14</sup>) In the acoustic case for example the summation law does not hold, so that such a simple mathematical formulation is impossible there.

<sup>15</sup>) The letters  $H$  and  $E$  used in the earlier article<sup>1</sup>) have been replaced by the modern notation  $B$  and  $S$ , while a subscript  $c$  is added to indicate that we are here concerned with the classical concept of brightness and with the standard eye-sensitivity curve. Since equation (2) indicates the classical brightness of a source of radiation as a function of purely physical properties, it is possible to measure classical brightnesses without using the eye as an instrument. This has led to the regular use of the concept of classical brightness even under conditions where the curve  $V_k(\lambda)$  is no longer valid. Under these conditions the classical concept of brightness will not satisfy the definition of equality.

<sup>16</sup>) H. H. Voet-Mogendorff, Diss. Amsterdam 1939.



above also follow from (1) by means of a definite choice of the function  $f$ . For the comparison light  $B$  is everywhere proportional to the energy radiated and the new concept of brightness coincides with the classical. From this it follows that for ranges of brightness in which  $V(\lambda)$  does not change with the brightness,  $B$  must always increase proportionally with the integral. If a field of vision of several degrees is chosen, this proportionality occurs in the region of pure cone vision (great brightnesses) and the region of pure rod vision (very low brightnesses).

In the first case (1) passes over into the classical definition (2), while in the second case

$$B_s = C_s \int S(\lambda) V_s(\lambda) d\lambda, \dots (4)$$

If we confine ourselves to these two limiting cases we can calculate directly from the classical brightness  $B_c$  the new brightness (subjective brightness, Purkyne brightness, "Dunkelleuchtdichte"), by dividing formulae (2) and (4) by each other:

$$\frac{B_s}{B_c} = \frac{C_s \int S(\lambda) V_s(\lambda) d\lambda}{C_c \int S(\lambda) V_k(\lambda) d\lambda} \dots (5)$$

The ratio  $C_s/C_c$  is found by substituting in (5) the spectral distribution  $S_0(\lambda)$  of the comparison light, for which  $B_s$  must equal  $B_c$ . We find the general formula:

$$\frac{C_s}{C_c} = \frac{\int S_0(\lambda) V_k(\lambda) d\lambda}{\int S_0(\lambda) V_s(\lambda) d\lambda} \dots (6)$$

If we choose for the comparison light a spectral colour with the wavelength  $\lambda_0$  then (6) passes over into:

$$\frac{C_s}{C_c} = \frac{V_k(\lambda_0)}{V_s(\lambda_0)} \dots (7)$$

From this formula it may be seen especially

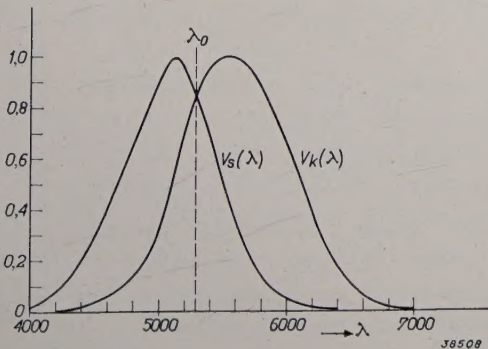


Fig. 1. Eye-sensitivity curves for fields of vision of several degrees in diameter.  $V_k$  holds for great brightnesses (standard eye-sensitivity curve),  $V_s$  holds for very low brightnesses (region of pure rod vision). The two curves intersect at  $\lambda_0 = 5290 \text{ \AA}$ .

that  $C_s = C_c$  when as comparison light the spectral colour is chosen of the wave length  $\lambda_4 = 5290 \text{ \AA}$ , where the curves  $V_k(\lambda)$  and  $V_s(\lambda)$  intersect each other (see fig. 1). For the different kinds of comparison light mentioned above formulae (6) and (7) give the following results:

Name of the concept of brightness	Comparison light	$C_s/C_c$
Subjective brightness	$\lambda_0 = 5350 \text{ \AA}$	1.20
"Dunkelleuchtdichte"	black radiation $2360^\circ \text{K}$	2.18
Definition with $\lambda_0 = 5290 \text{ \AA}$	$\lambda_0 = 5290 \text{ \AA}$	1.00

Fig. 2 shows schematically the shape of the function  $f$ . The brightness  $B$  and the expression  $\int S(\lambda) V(\lambda) d\lambda$  are here considered to be plotted in the same logarithmic scale. The Purkyne brightness (1) then gives an entirely linear relation.

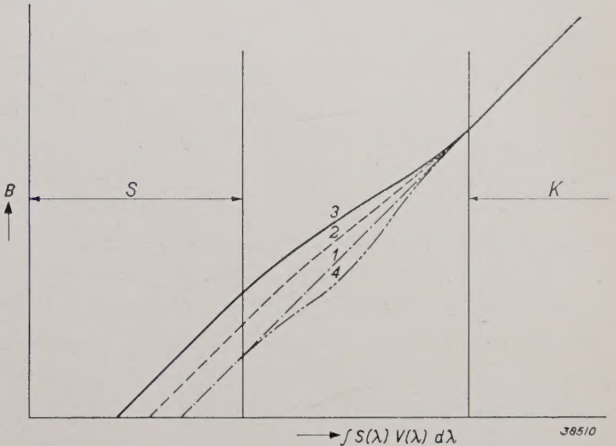


Fig. 2. Graphical representation of the relation given in formula (1) between the brightness  $B$  and the expression  $\int S(\lambda) V(\lambda) d\lambda$ .  
1 Purkyne brightness (straight line).  
2 Subjective brightness } (two straight sections not in the same straight line joined by a curve).  
3 "Dunkelleuchtdichte" }  
4 Brightness definition with  $\lambda_0 = 5290 \text{ \AA}$  as comparison light (two straight sections in the same line).  
 $S$  gives the brightness range within which there is pure rod vision,  $K$  the region in which pure cone vision occurs.  $B$  and the integral are considered to be plotted on a logarithmic scale.

The subjective brightness (2), the "Dunkelleuchtdichte" (3) and the concept of brightness based on the comparison wave length  $\lambda_0 = 5290 \text{ \AA}$  (4) give two straight sections, joined by a curved line. Only the left-hand section of the "Dunkelleuchtdichte" may be used; in the case of curve (4) the two straight sections are an extension of each other.

It is necessary to find new names for the units of the new concepts; particularly for the definitions with comparison lights it is quite possible to say, for instance "the classical brightness of a plane is



$3 \times 10^{-4}$  c.p./cm<sup>2</sup>, the subjective brightness only  $2 \times 10^{-4}$  c.p./cm<sup>2</sup>, which expresses the fact that the plane appears as bright as one which radiates monochromatic light of 5350 Å and possesses a brightness of  $2 \times 10^{-4}$  c.p./cm<sup>2</sup>. For practical reasons, however, a new name has been introduced in Germany for the unit of "Dunkelleuchtdichte", namely the "Skot" instead of  $10^{-7}\pi^{-1}$  stilb.

### Visibility of point sources of light

For light spots observed at an angle of several degrees the summation law and the eye-sensitivity curve  $V_s(\lambda)$  are found to retain their validity even at extremely low brightnesses in the neighbourhood of the threshold of the brightness observation, and the subjective brightness (or the "Dunkelleuchtdichte") at which the threshold value is reached is the same for all colours.

In blackout problems, however, one is concerned not only with the threshold value for large spots of light, but even more with the limits of visibility for light spots of extremely small dimensions. When the spots are sufficiently small their cross section is of no concern, and the visibility is determined exclusively by the amount and the colour of the light cast on the eye of the observer<sup>17)</sup>, i.e. of the intensity of illumination  $E$  on the eye of the observer. For this concept ("Punkthelle", "éclat apparente") we shall use the term "eye illumination". If by  $E_c$  one means the value of the classical concept of illumination intensity (based upon the standard eye-sensitivity curve  $V_k(\lambda)$ ), it is clear that the threshold value  $E_c$  will not be the same for all colours. By analogy with formula (5) which shows how it is possible to calculate the value  $B_s$  of the new concept of brightness (for instance, the "Dunkelleuchtdichte") which has the same threshold value for all colours for large light spots from the classical brightness  $B_c$ , it seems obvious to assume that from the classical illumination intensity  $E_c$ , with the help of an analogous formula

$$\frac{E_s}{E_c} = \frac{C_s \int S(\lambda) V_s(\lambda) d\lambda}{C_c \int S(\lambda) V_k(\lambda) d\lambda} \dots (8)$$

we should be able to calculate a "Dunkelbeleuchtungsstärke" at the eye which would then have to have the same value for all colours at the threshold of visibility.

In the modern German literature this assumption is accepted as obviously correct. For the unit of the new concept of intensity of illumination the name "Nox" instead of millilux is used.

On the basis of certain experimental data we shall now test the above assumption about the visibility of light points. In table I for six different kinds of light, all of which fall within a fairly narrow spectral region, the ratio is given between the threshold value of the eye illumination for coloured light ( $E'$ ) and ordinary electric light ( $E$ ).

Table I

Colour	Authors	I	II	III
		$E'_c : E_c$	$E'_s : E_s$	$\overline{E'_s} : \overline{E_s}$
red	Langmuir	16,8	0,80	1,75 (0%)
yellow (Na)	Weigel	4,1	0,83 <sup>5</sup>	1,56 (—11%)
yellow (Na)	Bouma	5,3	1,08	2,02 (+15%)
green	Langmuir	1,00	1,59	1,70 (—3%)
blue	Langmuir	0,310	4,25	1,92 (+10%)
blue	Arndt	0,258	3,44	1,54 (—12%)

Column I gives the ratio of the classical illumination intensities  $E'_c : E_c$ . As was to be expected this ratio depends very much upon the colour chosen. Column II gives the ratio between the "Dunkelbeleuchtungsstärken" calculated according to formula (8)  $E'_s : E_s$ . According to the assumption made the value 1.00 should be obtained for all colours. It is clear, however, that there are large deviations, especially in the blue, from which it is evident that the assumption referred to is quite incorrect.

Since the observation of points of light is quite a different problem from the observation of large spots of light, this is not surprising, and the question arises whether better agreement could be obtained by replacing the function  $V_s(\lambda)$  in (8) by another function  $\overline{V}_s(\lambda)$ , which is better adapted to the altered conditions of observation. In column III the ratios  $\overline{E'_s} : \overline{E_s}$  are given which are found when  $\overline{V}_s(\lambda)$  is so chosen that the threshold values become as nearly as possible equal for the different coloured lights.

The striking result is obtained that the ratio  $\overline{E'_s} : \overline{E_s}$  becomes reasonably constant, but that its average value is greater than unity, namely 1.75 (the deviations from this average value are given in parenthesis in the table). This means that for point sources of light the following assumption analogous to the summation law cannot be correct:

"When a radiation with the energy  $S_1$  (spectral distribution 1) and a radiation with the energy  $S_2$  (spectral distribution 2) both represent a threshold value, this is also true of the case of the mixture  $aS_1 + (1-a)S_2$ ".

<sup>17)</sup> Philips techn. Rev., 4, 15, 1939.



## Practical conclusions and applications to blackout problems

We shall now give a few applications of the above results to blackout problems.

### a) The use of the concepts "Dunkelleuchtdichte" and "Dunkelbeleuchtungsstärke"

From the above the conclusion may be drawn that for the road user who observes a large field of vision at low illumination intensities the brightness measured in "Skot" is a good measure of the visibility, but that for an air pilot, who will usually have to observe objects in the shape of points, the visibility is certainly not connected unambiguously with the "Nox" value of the eye illumination. In the case of light spots of at least several degrees in diameter, therefore, the introduction of the new concepts of brightness facilitates the calculation of the threshold value, but this is not true in the case of light points. If for example one considers the last colour mentioned in table I, then when the classical eye illumination (column I) is used it must be stated that the threshold value expressed in lux for white light is about 3.9 times that for blue light, while when the "Dunkelbeleuchtungsstärke" according to column II is used, it must be stated that the threshold value expressed in "Nox" for blue light is about 3.4 times that for white light. It is clear that in this case the introduction of the new concepts has brought no advantage at all.

The deviations observed in column II of table I may also be formulated in the following way: for the air pilot who observes points of light the threshold value for blue light lies considerably higher than would be expected, or, for blue light the Purkyne effect offers the air pilot much smaller advantages than the road user. Such a difference may be considered as an advantage in the use of blue light, to which statement it must immediately be added that this holds only for the case in which the blue light is used exclusively to make possible the general orientation of the road user. In all cases where it is a question of the recognition by the road user of definite signs (for instance letters as an indication of shelters) blue light has important disadvantages, since visual acuity is very low with this light.

### b) Estimation of the threshold value for coloured point sources of light

In order to reach an estimation of the threshold value for coloured point sources of light in spite of the proven unsuitability of the concept of "Dunkel-

beleuchtungsstärke" and the non-validity of the summation law, the experimental data of table I have been put into the form shown in fig. 3. In the horizontal direction the ratio of the threshold values of the classical eye illumination ( $E_c'$  for coloured light,  $E_c$  for ordinary electric light) has been plotted, as it would be calculated on the assumption that the "Dunkelbeleuchtungsstärke" or the subjective brightness was here a good measure of the visibility, in other words on the assumption that  $E_s' : E_s$  were equal to unity.

In the vertical direction the experimental values of  $E_c' : E_c$  are plotted. The values found are joined by the full drawn line. The figure is valid only for light sources which emit the greatest part of their light in a relatively narrow spectral region. In order to determine the threshold value of such a light source,  $(E_c'/E_c)_{\text{theor.}}$  is first calculated according to the formula:

$$E_c' : E_c = \frac{E_s}{E_c} : \frac{E_s'}{E_c'},$$

where the two quotients on the right-hand side can be determined with the help of equation (8). Then with the help of fig. 3, one finds the corresponding value of  $(E_c'/E_c)_{\text{exp.}}$

For light sources whose radiation is distributed over the whole spectrum fig.3 is not valid, and no conclusions can be drawn. An experimental determination will be necessary here. For small spots of light, which may not yet be considered as point sources (between about  $2'$  and  $1^\circ$ ), the full drawn curve of fig. 3 must be replaced by another curve which passes between the full line and the dotted straight line, and is closer to the latter the larger the light spot.

### c) The use of the filter method for blackout purposes

The filter method consists in the combination of sources of coloured light and coloured glass window panes such that while the window pane transmits a large percentage of daylight it absorbs almost all of the coloured artificial light<sup>18)</sup>. When this method is used the small percentage of light which reaches the outside is usually of a pronounced colour, so that care must be taken in calculating the threshold value.

If sodium light is combined with green window panes, the light emitted contains practically only the yellow sodium lines. From table I it may be seen that for this colour the threshold value of the eye illumination lies about 4.7 times as high as

<sup>18)</sup> Philips techn. Rev., 5, 93, 1940.



that for ordinary electric light. This factor cannot, however, be completely taken into account, since sodium light has such a striking colour that the correction factor  $N$  which must be introduced in

If electric lamps in orange-yellow glass bulbs are combined with green windows, it may be calculated from the transmission curve of Matthews and van Liempt<sup>18)</sup> (fig. 2) that  $(E'_c : E_c)_{\text{theor.}} = 0.62$ .

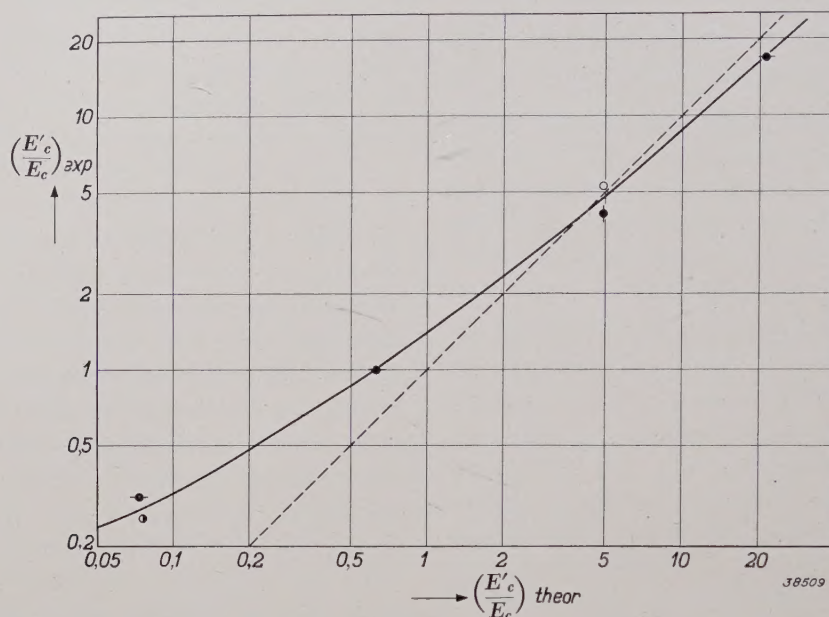


Fig. 3. The determination of the threshold values for coloured point sources of light. The ratio  $(E'_c : E_c)_{\text{theor.}}$  is first calculated from the threshold values of the eye illumination for coloured light and for ordinary electric light, assuming that for different colours the same number of "Nox" is required. With the help of the full drawn curve the true ratio  $(E'_c : E_c)_{\text{exp.}}$  of the threshold values is determined from this. The method may only be used for sources of light which emit the greatest part of their radiation in a narrow spectral region.

order to obtain from the laboratory values the practical threshold values<sup>17)</sup> must undoubtedly be lower than for white light. The actual ratio of the threshold values of the eye illumination will therefore be smaller than 4.7, but still always greater than unity, so that we remain on the safe side if in our calculations we treat the transmitted light as if it were white light.

Since here also the greatest part of the light is emitted in a relatively narrow spectral region, we may correct this value with the help of fig. 3, and we find  $(E'_c : E_c)_{\text{exp.}} = 0.99$ . Since with this colour we must expect for the correction factor  $N$  a value which does not deviate greatly from that for white light, it is also permissible here to neglect the influence of the colour of the light on the visibility.



## MAGNETIC OIL FILTERS

by J. A. HARINGX.

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Filters in which the oil is made to flow through a magnetic field have been constructed for the removal of particles of iron from circulating lubricating oil. In this article a description is given of such a filter which has in addition to a good efficiency in capturing iron particles many other advantages also, such as very low pressure loss in the oil circuit, no chance of the filter becoming clogged, ease in cleaning the filter when it is saturated. The action of this type of filter is studied. In ordinary use, where new particles of iron are continually entering the oil due to wear on the parts of the machine lubricated, an equilibrium is found to occur which is characterized by a certain residual concentration of iron in the oil. As the filter gradually fills up, the equilibrium is shifted to higher concentrations. If a certain limit is fixed for the residual concentration, the time can be calculated which may elapse before the filter must be cleaned. By means of a series of tests the necessary data have been obtained. The results are elaborated into a graph in which the influence of the viscosity of the oil is taken into account.

In the pressure lubrication of bearings, gears, etc. it is very important to keep the circulating oil as clean as possible. If the oil is contaminated, for instance by fine particles of iron from the wear on machine parts which rub against each other, the formation of a continuous film of oil is not only prevented, so that the lubricating power decreases, but in addition the particles cause extra wear by grinding against the parts in question. Furthermore finely divided metals, especially iron, by catalytic action, cause acidification of the oil, which is also a disadvantage for the lubricating properties.

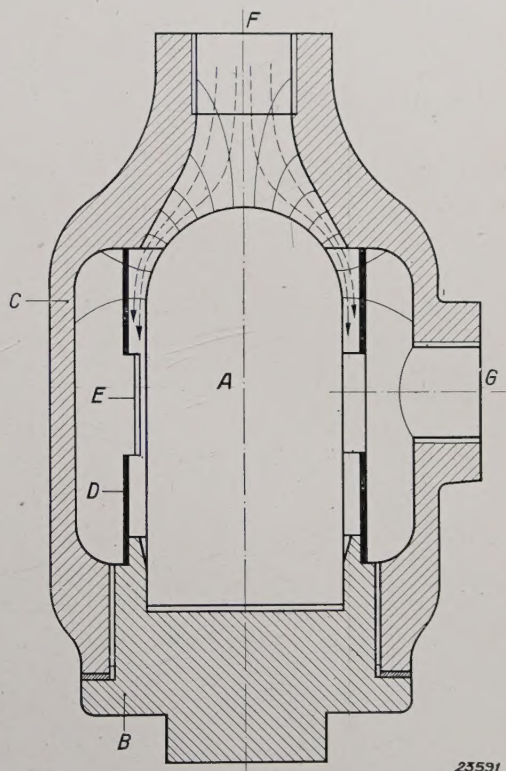
In addition to the old textile or copper gauze filters which are used for the purification of circulating lubricating oil, magnetic filters have recently become more and more common. A magnetic filter of simple construction was described several years ago in this periodical<sup>1)</sup>, while several possibilities of application were also discussed at that time. In the meantime a new filter of improved construction (type No. 7 715) has been developed<sup>2)</sup> and a series of investigations on the functioning of these oil filters has been carried out. In the following article the filter and the tests referred to are described.

### The construction

In order to be able to explain the essential features of the new construction, the simplest way is perhaps to consider first the construction described previously. This is shown in *fig. 1*. In an iron housing a permanent magnet is so fastened that a magnetic circuit with a ring-shaped air gap is formed. The oil to be purified flows through this air gap. The construction is such that the stream lines of the oil

and the lines along which the particles of iron are drawn to the magnet intersect each other at the smallest possible angles; by this means a relatively small lateral acceleration of the iron particles in the oil is already enough to take them out of the current and to the surface of the magnet.

Although this filter had a very satisfactory cleaning power and was able to take up large quantities of iron filings, in practice various undesired



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Fig. 1. Magnetic oil filter of the old type. The magnetic circuit is formed by the permanent magnet *A*, the iron cap *B* and the iron housing *C*. The oil entering at *F* flows through the air gap between *A* and *C* past the holes *E* in the copper cylinder *D* and leaves the filter at *G*. The stream lines of the oil are indicated by broken lines, the magnetic lines of force are drawn as full lines.

<sup>1)</sup> L. H. de Langen, Philips techn. Rev., **2**, 295, 1937.

<sup>2)</sup> J. B. Aninga, Polytechn. Wbl., **33**, 86, 1939.



phenomena were found to occur when the filter was approaching the limit of its capacity of retention. Due to the deposition of the iron the available opening for the flow of oil, which in connection with securing a strong magnetic field could not be made all too wide, became narrower and narrower, with the result that on the one hand a considerable pressure loss occurred in the filter, while on the other

up the requirement the above-mentioned disadvantages could be avoided.

The new construction is shown in *fig. 2a* and *b*. The magnetic circuit is built up of a cylindrical magnet provided with two disc-shaped pole pieces and five soft iron rings placed around the magnet one above the other at small distances apart. In the intermediate spaces — the air gaps — strong

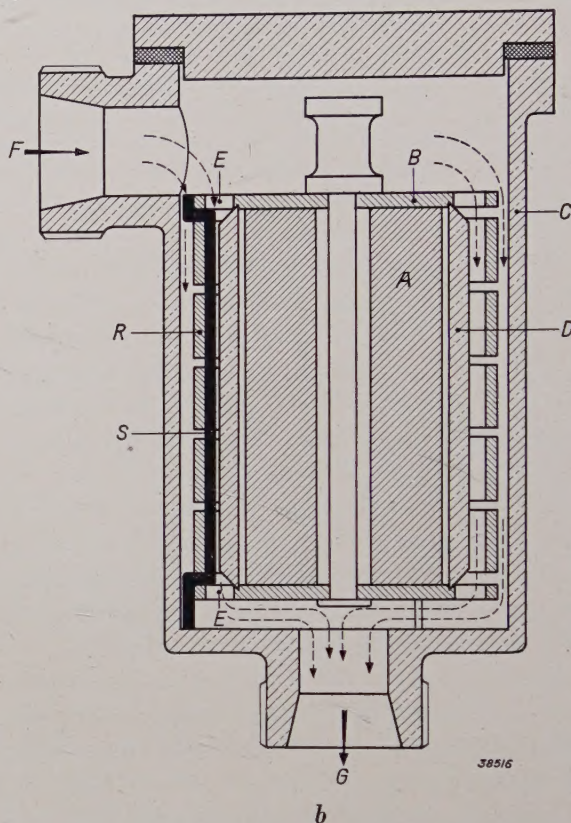
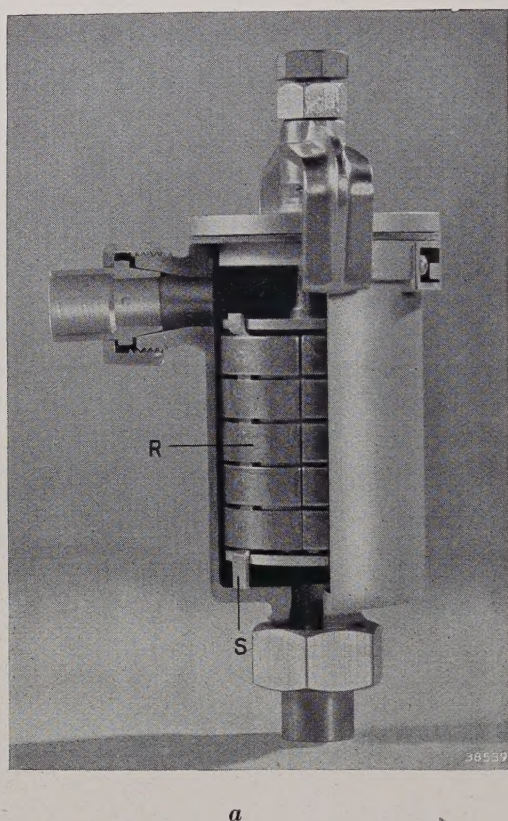


Fig. 2. Magnetic filter of new construction. In the cross section diagram (*b*) may be seen the permanent magnet *A* which with the pole pieces *B* and the five rings *R* forms the magnetic circuit. The oil entering at *F* flows via the holes *E* in the pole pieces past the inside and outside walls of the set of rings, and thus not through, but along the air gaps. The housing *C* and the cylinder *D* are of non-magnetic material. The oil leaves the filter at *G*. The rings are in two sections, as may clearly be seen in the photograph of the opened filter (*a*), and are held in place by brass strips *S* welded to them. The whole magnetic system also rests upon the ends of these strips. After unscrewing the cover of the housing the magnet system can be drawn out of it by means of the handle provided.

hand a wisp of the deposited material was sometimes torn away by the oil and passed into circulation again. Furthermore the cleaning of the filter when full was fairly difficult, since the magnet retains the deposit very firmly, a fact with which it may not be reproached since that is its function.

In the new construction the idea that the stream lines of the oil and the lines of attraction of the iron particles should intersect at only small angles has been given up. Considering the high strengths of the magnetic fields which can be obtained with the magnet steels available at present this condition is indeed no longer so important, and by giving

magnetic fields occur. The oil now, however, flows not through but along the inside and outside of the air gaps, as illustrated in *fig. 3*. It may be seen that it is actually the spread lines of force of the magnetic field which draw the iron particles floating in the oil out of their original paths and cause them to enter the air gaps. Due to the fact that the iron particles are now deposited outside the stream of oil, the tearing off of bits of the deposit has been made impossible, while at the same time no appreciable increase in loss in pressure can occur as the filter gradually fills up. In the tests to be described below the pressure loss, with a flow



of 500 l/hr and a viscosity of the oil of 35 cp<sup>3</sup>), was found to increase during use only from 0.05 atm. in empty condition to 0.10 atm. in practically full condition of the filter. A pressure loss of this small magnitude is of no practical significance; at least,

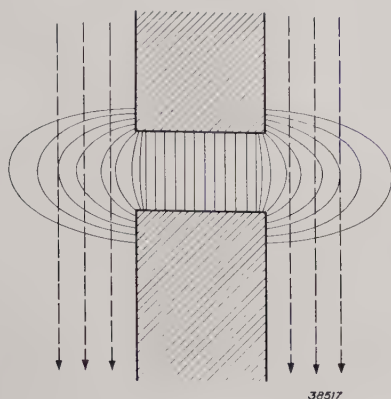


Fig. 3. Sketch of the stream lines (broken lines) and the magnetic lines of force (full lines) in the neighbourhood of one of the air gaps.

compared with the pressure losses at this rate of flow in textile filters, it may be neglected. The behaviour of the pressure loss for other values of the viscosity may be seen in *fig. 4* where curve *a* holds for the empty state and curve *b* for the practically full state of the filter.

The cleaning of the filled filter could also be made very much easier with the new construction than with the old. For this purpose each of the rings is made in two sections which are held free in position by brass strips. For cleaning, the whole magnet system is removed from the housing and the halves of the rings can easily be drawn off the magnet. Due to the fact that the rings are now no longer

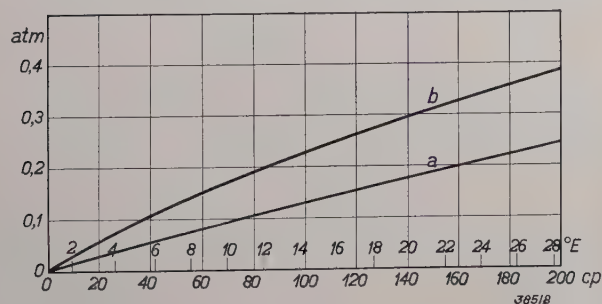


Fig. 4. Pressure loss in the magnetic filter with a rate of flow of 500 l/hr as a function of the viscosity of the oil in cp and °E, respectively. Curve *a* holds for the clean filter, *b* for the filter filled with 7 g of deposit.

<sup>3</sup>) A liquid has a viscosity of 1 poise (p) or 100 centipoise (cp), when a shearing stress of 1 dyne/cm<sup>2</sup> is necessary to maintain in the liquid a velocity gradient of 1(cm/sec)/cm. At 20°C for instance water has a viscosity of about 1 cp, rape oil one of about 100 cp. Technically the viscosity is usually given on an empirical scale (Engler degrees, °E) which is also indicated on the abscissa of *fig. 4*. The recalculation into cp is impossible with strict accuracy.

magnetic the deposit can be rinsed off without difficulty with some liquid or other such as petrol, trichlorethane or the like. To prevent particles of iron being deposited on the permanent magnet itself a non-magnetic cylinder is introduced between the magnet and the rings (*D* in *fig. 2b*).

The quantity of deposit which the filter can take up is proportional to the total volume of the air gaps. The larger this volume is, however, the lower the magnetic field strength which can be obtained. A suitable compromise had therefore to be found. With the chosen dimensions of the air gaps (1.5 mm long, 275 mm<sup>2</sup> cross section, thus a total volume of  $6 \times 415 =$  about 2500 mm<sup>3</sup>) a total of about 10 grams of iron filings can be taken up, while a magnetic field strength of 8000 gauss is obtained.

A second compromise was necessary as to the cross section of the channel of flow for the oil. This channel must be made narrow so that the iron particles to be captured pass as close as possible along the air gaps. On the other hand less oil per hour can be dealt with when the channel is narrow, if the velocity of flow of the oil is not to be increased,

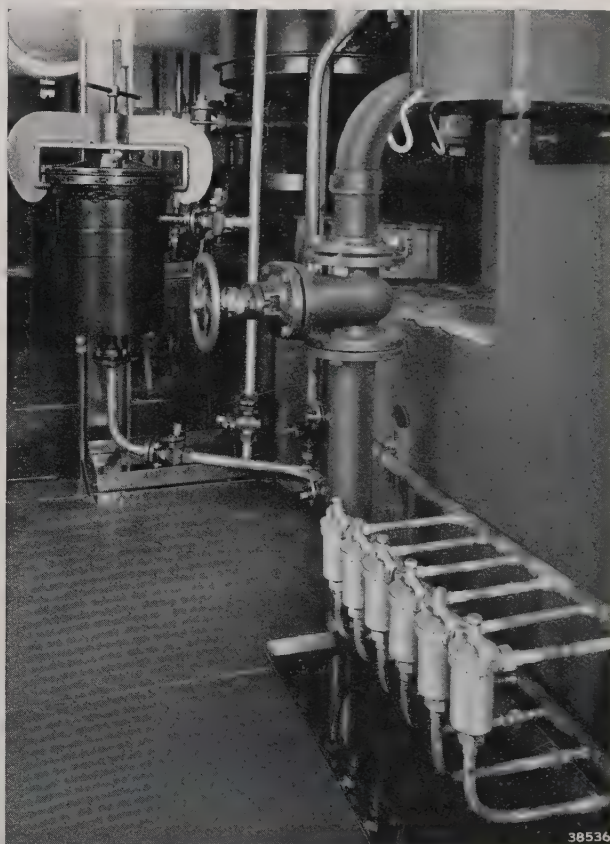


Fig. 5. Part of the lubrication system of an 800 h.p. Diesel engine. In the foreground six magnetic filters may be seen which are connected in parallel in the oil line. In the cylindrical tank on the left in the background there is a copper gauze filter which serves to remove coarse impurities from the oil.



thereby increasing the chance that some iron particles escape being captured by the filter. The dimensions finally chosen (cross section 550 mm<sup>2</sup>) make it possible to deal with 500 litres of oil per hour. In special cases where greater flow capacities are necessary several filters may be connected in parallel in the oil line. An example of such an installation with six filters is reproduced in *fig. 5*.

#### Further consideration of the action of the filter

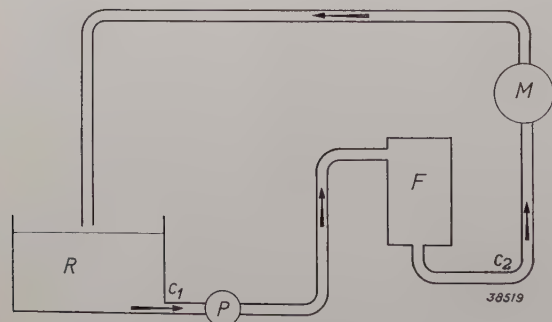
When the magnetic filter is installed in a closed system in which oil circulates which is very much contaminated by iron, the oil is seen to become clear only gradually, for instance within fifteen minutes or a half hour. This shows that all the iron particles are not caught by the filter at once as they pass it, but a certain fraction only each time: the oil must pass repeatedly through the filter before the concentration of iron falls practically to zero. If a certain quantity of new iron is continually formed in the system, the concentration is found never to fall below a certain value.

As we shall see this limiting value of the concentration is indeed very much lower than the concentration which would be present in the oil without the use of the filter; but the residual concentrations cannot in general be entirely neglected. It was therefore desirable to investigate the action of the filter more closely.

#### The residual concentration

*Fig. 6* shows diagrammatically the situation in which the filter is used. The lubricating oil, which is kept in circulation by a pump, comes from a reservoir, passes first through the filter, then along the part of the machine to be lubricated and finally returns to the reservoir.

Suppose that the oil in the reservoir has an iron concentration of  $c_1$ . If  $V$  is the amount of oil flowing through the filter per unit of time, the filter must



*Fig. 6.* Diagram of a lubricating system with oil circulation. *R* reservoir, *P* oil pump, *F* magnetic filter, *M* part of the machine to be lubricated. The oil has an iron concentration of  $c_1$  preceding the filter and iron concentration of  $c_2$  after passing it.

deal with an amount of iron particles equal to  $c_1 V$  per unit of time. As a first approximation we may now assume that the same fraction of this quantity brought to the filter is always retained by it. The amount of deposit already caught by the filter, which we call  $G$ , thus grows per unit of time by the amount

$$\Delta G = \gamma c_1 V, \quad \dots \dots \dots (1)$$

where  $\gamma$  is a factor smaller than unity which we shall in the future designate as the retention coefficient.

If  $Q$  is the amount of new sediment formed per unit of time,  $Q$  may be larger or smaller than  $\Delta G$  or equal to  $\Delta G$ . When  $Q > \Delta G$  more sediment is introduced into the circulating oil than is removed from it. The concentration  $c_1$  of the iron will therefore gradually increase, at the same time, however, according to (1) the amount captured per unit of time,  $\Delta G$ , also becomes larger until the addition of iron to and its removal from the oil are just balanced, *i.e.* until  $Q = \Delta G$ . In the same way when  $Q < \Delta G$  the concentration  $c_1$  will gradually fall until  $\Delta G$  has also fallen so far that  $Q = \Delta G$ . It is therefore clear that in any case after some time a condition of equilibrium will be established where

$$\Delta G = Q = \gamma c_1 V \quad \dots \dots \dots (2)$$

From this we can calculate the residual concentration  $c_1$  which may be expected. We must, however, keep in mind that the concentrations of the iron preceding and following the filter are not equal. Of the amount of iron entering the filter  $c_1 V$  a part  $\gamma c_1 V$  is held by it. Thus only the amount  $(1-\gamma)c_1 V$  leaves the filter, *i.e.* the concentration  $c_2$  of the iron in the oil conducted to the part of the machine to be lubricated amounts to

$$c_2 = (1-\gamma) c_1 \quad \dots \dots \dots (3)$$

From this with (2) it follows that

$$c_2 = \frac{1-\gamma}{\gamma} \frac{Q}{V} \quad \dots \dots \dots (4)$$

When the filter has functioned during a time  $t$  in the equilibrium state, with a constant addition ( $Q$ ) of new sediment to the oil, it has taken up a total quantity of iron

$$G = Q \cdot t \quad \dots \dots \dots (5)$$

To give a numerical example: suppose that in the machine  $Q = 5$  mg/hr of new iron filings are formed and that the retention coefficient has the apparently very low value of  $\gamma = 0.01$ . If we substitute for  $V$  the above-mentioned value of 500 l/hr,  $c_2$  becomes only 1 mg/l.



With a retaining capacity of  $G_{\max} = 7$  grams, according to (5) the filter must be cleaned after 1400 hours of use. Without the filter after 1400 hours, when a total of 50 litres of oil take part in the circulation there would be an iron concentration of 140 mg/l<sup>4</sup>).

#### *Further consideration of the retention coefficient*

How large is the retention coefficient  $\gamma$  of the filter described? It is impossible to give an immediate answer to this question since the above assumption of a constant value of  $\gamma$  is by no means justified. The retention coefficient depends upon a number of factors, the chief of which are: the degree  $G$  to which the filter is filled, the iron concentration  $c_1$  in the oil, the viscosity of the oil, the rate of flow and the size of the iron particles. The last three factors remain approximately unchanged during use of the filter, or at least they may be kept constant in the experiments to be described in the following. This is, however, not true of the first two, so that we must extend our consideration of this point somewhat. We shall first discuss the influence of the degree  $G$  to which the filter is filled.

When the filter is entirely filled with the deposit  $\gamma$  must in any case fall to zero. This transition to zero does not take place abruptly, which is understandable when it is kept in mind that the magnetic field itself is influenced by the iron particles taken up. The field strength will become greater due to the decrease in the magnetic resistance of the air gaps. At the same time, however, the spreading, which is just what we must have, will become less. This last effect is found to dominate and for the

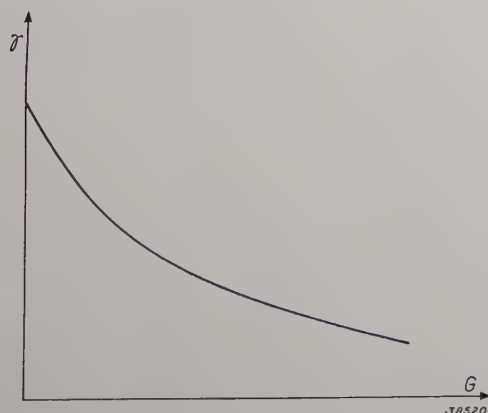


Fig. 7. Approximate form of the curve showing the variation of the retention coefficient  $\gamma$  with the degree of filling  $G$  of the filter.

<sup>4</sup>) In general before such high concentrations are reached the oil will have to be renewed.

behaviour of  $\gamma$  with  $G$  a curve of the form sketched in fig. 7 is obtained.

The result of this variation of  $\gamma$  with  $G$  is that, strictly speaking, we may no longer speak of a definite equilibrium condition: during use, as the filter slowly fills up,  $\gamma$  will decrease slowly and the iron concentration in the oil will gradually increase. Nevertheless upon closer consideration it is found that when  $\gamma$  does not vary too much with  $G$ , a condition which is always satisfied in use, the successive states of the system may still be conceived of as a gradually shifting equilibrium. This means that we may continue to use equation (4) for the calculation of  $c_2$ , with a different value of  $\gamma$  at every moment corresponding to the degree of filling  $G$  of the filter at that moment.

If the requirement is made that the residual concentration  $c_2$  of the iron may not exceed a certain value, it includes the requirement that the filter must be cleaned, not when it is full, but sooner, namely when the retention coefficient has fallen to that value of  $\gamma$  which according to (4) corresponds to the permissible value of  $c_2$ . From a graph like that of fig. 7 one may then read off at what degree of filling  $G$  the filter should be cleaned, and from equation (5) the time can be calculated which is necessary to reach this degree. Equation (5) is no longer exactly valid, since due to the slow shift of the equilibrium a certain amount of the newly formed sediment is used in increasing the iron concentration of the oil. Nevertheless, considering the low concentration and the small quantities of oil which are usually used for the circulation, the deviation from equation (5) is only slight.

The second factor which varies during use and which affects the retention coefficient is the concentration  $c_1$  itself. Its effect is easily understood qualitatively. The diagram of the lines of force sketched in fig. 3 will be somewhat altered when the magnetic resistance is decreased on both sides of the air gap proper by the presence of a large number of iron particles. The "spreading" increases as it were with the iron concentration of the oil, and at the same time the attractive effect of the field also increases. This effect is found to be quite considerable in magnitude.

The fact that  $\gamma$  depends upon  $c_1$  can be taken into account in quite the same way as above. For every state characterized by a definite value of  $G$  and of  $c_1$  there is a certain value of the retention coefficient  $\gamma$ . If we represent this relation graphically by plotting  $\gamma$  as a function of  $G$  for a series of values of  $c_1$  (fig. 8), we can again read off the degree of filling  $G$  of the filter at which it must be cleaned,



when a given maximum residual concentration  $c_2$  is prescribed.  $\gamma$  can be calculated from  $c_2$  according

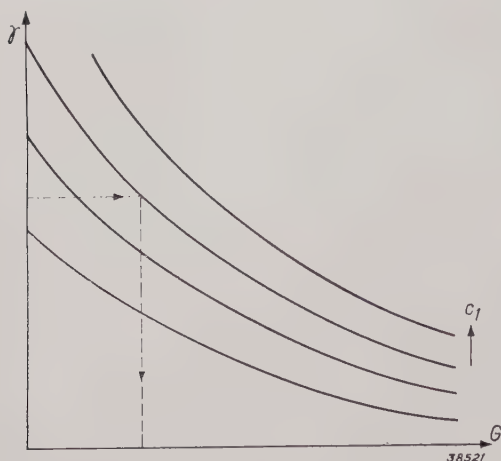


Fig. 8. Approximate form of the family of curves which indicates the retention coefficient  $\gamma$  as a function of the degree of filling  $G$  with as parameter the iron concentration  $c_1$  at the inlet to the filter. If a given residual concentration  $c_2$  is permissible at the outlet of the filter, with the aid of equations (4) and (3)  $\gamma$  and  $c_1$  can be calculated, and the permissible degree of filling can then be read off in this graph.

to (4) and  $c_1$  according to (3). We shall see below how this graph can be drawn somewhat more simply; the principle, however, remains the same.

### Experimental determination of the retention coefficient

It is now a question of studying more closely the relation  $\gamma (G; c_1)$  sketched in fig. 8 for our filter, which can only be done experimentally. The necessary experiments are preferably related as much as possible to practical conditions. One of the conditions should therefore be a constant and regularly distributed addition  $Q$  of iron particles to the oil, and in order to include the whole range of degrees of filling the experiment for each value of  $Q$  (with the corresponding series of concentrations  $c_1$  which are traversed) should be continued for a long time, as is apparent from the numerical example described above. In order to avoid this we have arranged the experiments somewhat differently. A certain quantity ( $H$ ) of iron filings was added once only to the circulating oil (total amount  $v$ ). In contrast to the usual case in which the iron concentration increases very slowly as the filter becomes full, we here have a fairly rapid decrease of the concentration, and the experiment is concluded in a few hours. The variation of the concentration  $c_1$  with the time is determined by measuring  $c_1$  chemically at specified intervals. The retention coefficient  $\gamma$  is then found as follows. At each moment the following is true:

$$G = H - vc_1 \dots \dots \dots (6)$$

The amount of iron  $G$  retained in the filter thus grows per unit of time by

$$\frac{dG}{dt} = -v \frac{dc_1}{dt}.$$

On the other hand, according to the definition of the retention coefficient (equation (1)) this growth was given by  $\gamma c_1 V$ . Therefore

$$\gamma c_1 V = -v \frac{dc_1}{dt},$$

$$\gamma = -\frac{v}{V} \frac{d \ln c_1}{dt} \dots \dots \dots (7)$$

Thus if we plot  $\ln c_1$  against  $t$ ,  $\gamma$  follows from the slope of the curve obtained.

In all experiments the maximum rate of flow of  $V = 500$  l/hr was used, while the viscosity of the oil, which varies very much with the temperature, was kept constant by placing the reservoir in a thermostat. The experiments were carried out with a very fine carbonyl iron powder which satisfactorily resembles the sediment most commonly formed in practice as far as shape and size of the particles (about 2 microns) are concerned. After the addition of the powder ( $G = 1$  to 10 g in  $v = 5$  litres of oil), the oil was first pumped through the system for several hours with no magnet in the filter, in order to distribute the iron evenly throughout the oil. Special care had to be taken that all the oil took part in the circulation, *i.e.* that there was no direct

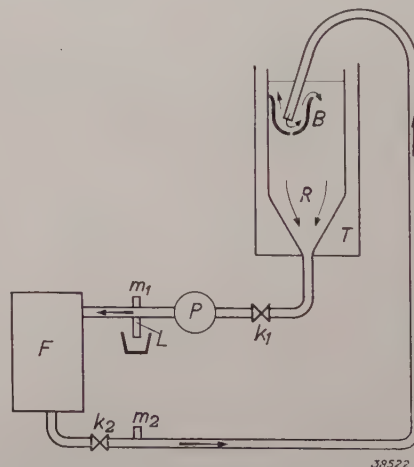


Fig. 9. Arrangement for the experiments for the determination of the retention coefficient.  $R$  reservoir,  $T$  thermostat,  $P$  tooth-wheel pump,  $F$  magnetic filter,  $k_1, k_2$  taps. In order to avoid a direct current in the reservoir between inlet and outlet, which would hinder the uniform distribution of the iron throughout the oil, the inlet ends in a vessel  $B$  over the edge of which the oil flows in all directions. There is a small hole in the bottom of the vessel to prevent any iron which has settled from being removed from the circulation. At specified intervals a small quantity of oil is tapped off through the line  $L$  and its iron concentration is determined. At  $m_1, m_2$  a differential manometer was connected which indicated the pressure loss in the filter.



current in the reservoir from inlet to outlet. Fig. 9 shows how this was accomplished.

The apparently so simple experiments still contained many stumbling blocks. While the circulation pump is getting under way, for example, there is great danger of the formation of small air bubbles in the oil, which later have no opportunity of escaping from the rapidly flowing oil and which upon passing through the filter may affect the deposition of iron particles considerably by their surface tension. By starting the pump as gradually as possible the formation of air bubbles is almost entirely avoided. Furthermore a small amount of iron powder is found to cling to the walls of the circulation system at various points, which results in the fact that the iron concentration is slightly lower than corresponding to equation (6). This can be taken into account by using a corrected value for  $v$  as well as for  $H$ . Further, the concentration  $c_1$  in equation (6) is actually an average value over the total quantity of oil, while the action of the filter is determined by the concentration at the inlet to the filter. A correction must also be applied for this by correlating the slope ( $\gamma$ ) found at point  $t$  in the  $\ln c_1-t$  curve with the average value of  $c_1$  over an adjacent time interval  $\tau$ ;  $\tau$  is here the time necessary for the total amount of oil to be pumped once around the system, in our case about 36 sec. Nevertheless, after all these corrections, the results still show considerable divergence, caused by the limited accuracy of the measurement of the concentration. This measurement is made by tapping from the oil line in fig. 9 a sample of 25 cc of oil, "igniting" the oil and determining the iron content of the residue.

Results of measurements

In the manner described three series of measurements were carried out in the first instance, in which 1 3/4, 5 and 10 g of iron powder, respectively, were added to the oil. In fig. 10 the variation of the measured concentration  $c_1$  is given as function of the time elapsed after the insertion of the magnet for these three cases (1, 2, 3). If we now consider for instance curve 2, for every point  $c_1$ ,  $t$  of the curve we can determine the values of  $\gamma$ ,  $c_2$  and  $G$

corresponding to that value of  $c_1$  according to equations (7), (3) and (6). In table I the result of this determination is given for a series of values of  $c_1$  for curve 2.

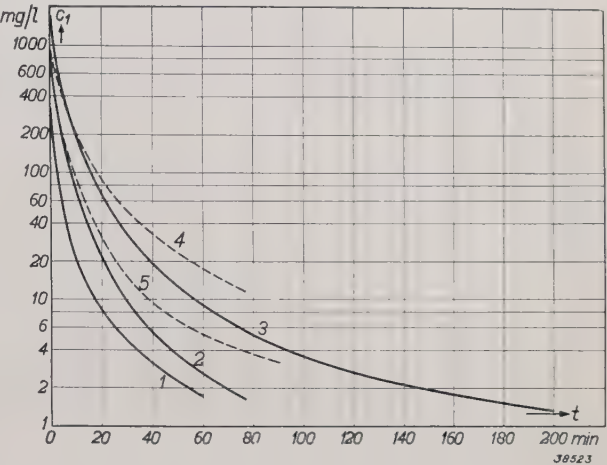


Fig. 10. Variation of the iron concentration  $c_1$  (in mg/l) with the time  $t$  (in min) in different experiments. At the beginning of each experiment quantities of iron were added to the oil:  $H = 1.75$  g (curve 1), 5 g (curve 2) and 10 g (curve 3).  $\ln c_1$  is plotted against  $t$  in order to find the retention coefficient  $\gamma$  by graphical differentiation according to equation (7). The broken line curves 4 and 5 with  $H = 5$  and 4 g, respectively, were recorded with a higher viscosity of the oil, namely with  $\eta = 85$  cp, while in curves 1, 2 and 3 the viscosity was 35 cp.

According to the discussion above, for every value of the parameter  $c_1$  we would have to plot the retention coefficient  $\gamma$  as a function of  $G$ . Then on the graph obtained, beginning with  $\gamma$  and  $c_1$ , the permissible value of  $G$  could be read off. In the practical application, however, we must assume that the values of  $Q$  (the quantity of new sediment continuously formed in the part of the machine being lubricated) and of  $c_2$  (the permissible residual concentration) are given, and that  $\gamma$  and  $c_1$  must then be calculated with the help of (4) and (3). It

Table I

Elaboration of the series of measurements 2 and 5 (as examples). The rows with large numbers are derived directly from the curves 2 and 5, respectively in fig. 10, while the rows with small numbers are found by interpolation.

Experiment 2					Experiment 5					
$c_1$ mg/l	$\gamma$	$c_2$ mg/l	$G$ mg	$\frac{\gamma c_2 V}{1-\gamma}$ g	$c_1$ mg/l	$\gamma$	$c_2$ mg/l	$G$ mg	$\frac{\gamma c_2 V}{1-\gamma}$ g	$\frac{\gamma c_2 V \eta'}{1-\gamma \eta}$ g
786	0.260	580	0		753	0.152	638	0		
	0.237	500	830	77.6		0.141	500	1070	41.0	100
400	0.196	321	2410		400	0.126	350	1868		
	0.110	100	3853	6.16		0.081	100	3180	4.40	10.8
50	0.086	46	4187		50	0.0585	47	3450		
	0.068	20	4343	0.730		0.0415	20	3590	0.434	1.065
20	0.068	18.6	4350		20	0.0415	19.2	3594		
10	0.052	10	4402	0.274		0.0263	10	3640	0.135	0.331
5	0.036	5	4431	0.0931	10	0.0260	9.75	3641		
	0.0250	3	4443	0.0384		0.0123	5	3665	0.0311	0.0763
2	0.0172	2	4449	0.0174	3	0.0056	3	3675	0.0084	0.0206



is therefore simpler to make the graph in such a way that the given quantities, in addition to the required value of  $G$ , occur in it directly. In order to do this we choose  $c_2$  as parameter with the values of  $1\frac{1}{2}$ , 2, 3, 5, 10 ... mg/l, for example. In table I several of these values are filled in by intrapolation (by means of auxiliary graphs) and in each row the quantity

$$\frac{\gamma c_2 V}{1-\gamma} = Q \dots \dots \dots (8)$$

is calculated and indicated in the fifth column. (For the rate of flow  $V$  we always use the maximum value of 500 l/hr, with which the experiments were also performed). If we now plot this quantity as a function of  $G$  with  $c_2$  as a parameter, the required value of  $G$  can be read off directly from the given values of  $Q$  and  $c_2$ .

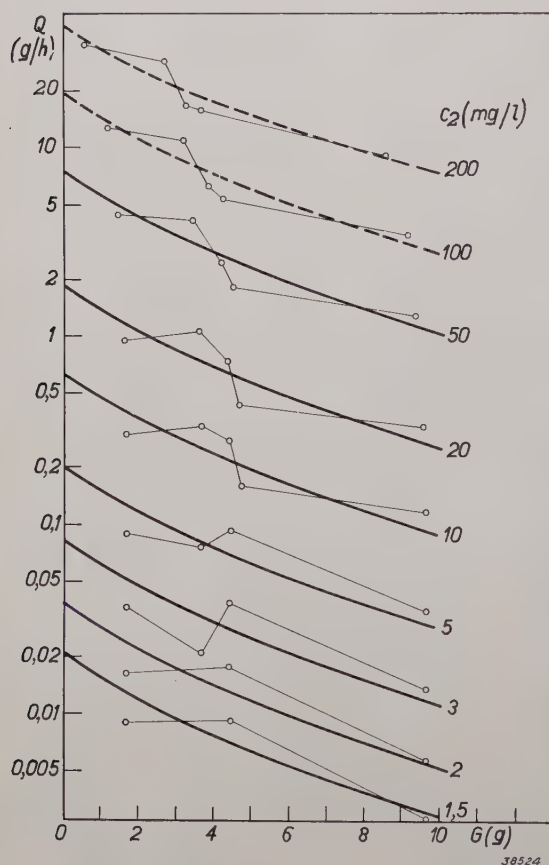


Fig. 11. The quantity  $\gamma c_2 V / (1-\gamma)$ , which is equal to  $Q$  in the equilibrium state of normal functioning, is here plotted as a function of  $G$  with  $c_2$  as a parameter. For every curve  $c_2$  there are three measured points available from the series of measurements 1, 2, 3, while two series of measurements 4, 5 carried out with a different viscosity of the oil provided two more points for every curve. (For the broken line curves the quantity  $\gamma c_2 V / (1-\gamma)$  here plotted may not be set equal to  $Q$ , since at such high concentrations the change in the quantity of iron present in the oil may no longer be neglected. The curves are, however, of scarcely any importance, and are here given only for the sake of fixing more completely the shape of the curves of the whole family).

For each curve for one value of  $c_2$ , each of the three series of measurements 1, 2, 3 gives one point (a pair of values of  $Q$  and  $G$ ), so that we would have to draw each curve with the aid of only three measured points. This would be quite difficult considering the fairly great divergences. Two other series of measurements (see below) performed by a slightly different method, however, provided two more points for every curve, so that the shape of the curves was somewhat better determined. The family of curves obtained is reproduced in fig. 11.

### The influence of the viscosity

While in the above in the theoretical as well as the experimental considerations we have only investigated the influence of the degree of filling  $G$  and the iron concentration  $c_1$  on the retention coefficient, we shall in conclusion also study the influence of the viscosity of the oil. The fact that the viscosity will influence the action of the magnetic filter is easily understood: the more viscous the liquid, the greater the force necessary to remove a particle from it. At a given rate of flow the magnetic field will capture the iron particles from a thick oil less well than from a thin oil.

As in our experiments, we may also assume that for normal use in a given installation, in which the working temperature and type of oil chosen are fixed, the oil will have a constant viscosity. A graph like fig. 11 can therefore be used, with a separate curve for each viscosity. Actually, however, there is a still simpler method.

The three series of measurements 1, 2, 3 in the experiments with the arrangement described were obtained with a single definite viscosity of the oil, namely  $\eta = \text{approx. } 35 \text{ cp}$ . Two additional experiments (4 and 5) were performed with a viscosity of  $\eta' = \text{approx. } 85 \text{ cp}$ . The variation of the measured concentration with the time for these latter cases, where  $H = 5$  and  $4 \text{ g}$ , respectively, is given as a broken line in fig. 10. It may immediately be seen that the retaining action of the filter is smaller since the iron concentration decreases more slowly. If, however, the retention coefficient  $\gamma$  as well as the values of  $c_2$ ,  $G$  and  $\gamma c_2 V / (1-\gamma) = Q'$  (see table I where this has been done for curve 5), are again calculated from the curves, and if again as above the points of a family of curves as in fig. 11 are determined from these values, these points are found to fit very well in the old family of curves when the values of their ordinates  $Q'$  are multiplied by the ratio of the viscosities  $\eta'/\eta = 85/35$ . The agreement — as far as may be expected with the fairly large divergences already mentioned — is so good that



we used the two new points obtained in this way for every  $c_2$  curve to draw a smooth curve through the points.

The result is therefore that the same family of curves is obtained if for experiments with the viscosity  $\eta'$  the quantity  $\eta'Q'$  is plotted and for experiments with the viscosity  $\eta$  the quantity  $Q\eta$ . We have therefore redrawn the graph of fig. 11 with this ordinate, see fig. 12, so that it may be used for any desired viscosity. At the same time a number of interpolated curves are given in this figure.

A numerical example will serve to illustrate the use of the graph. A large gear box in which about 25 mg of sediment is formed per hour (this apparently very high figure actually does occur in practice) is lubricated with 1 000 litres of oil per hour. Since the maximum capacity of flow of the magnetic filter here described is 500 l/hr, two filters in parallel must be installed in the oil circuit. At the working temperature the oil has a viscosity of 50 cp. The problem is to determine when the filters must be cleaned if the iron concentration may not exceed 2 mg/l (in general a considerably greater concentration will be permissible in a gear box).

Per filter the amount of new sediment introduced is  $Q = 12.5$  mg/hr. If in fig. 12 we intersect the horizontal line corresponding to  $0.0125 \times 50 = 0.625$  by the curve corresponding to  $c_2 = 2$  mg/l, then at the abscissa of the point of intersection it may be read off that each filter may become filled to a weight of  $G = 3$  grams of iron particles. The time necessary for this is  $3/0.0125 = 240$  working hours. After that time the filters must be cleaned. (If one waits for instance 600 instead of 240 hours  $G$  becomes 0.5 g and according to fig. 12 the residual concentration has still only increased to 3 mg/l).

If it were desired in the above case to keep the iron concentration below 2 mg/l without the use of filters, then per working hour  $25/2 = 12.5$  litres of fresh oil would have to be added. This illustrates very clearly the advantage of the use of such magnetic filters.

In conclusion a few words may be said about the permissible value of the iron concentration  $c_2$  (residual concentration). This quantity, like the amount of sediment  $Q$ , will vary very much according to the nature of the part of the machine lubricated; with accurately finished axles and bearings  $c_2$  must be small, for instance not more than several mg/l. If at the same time a high value

of  $Q$  and/or  $\eta$  occurs it might be that the corresponding horizontal line in fig. 12 lies in its entirety higher than curve  $c_2$ . This would mean that the retention coefficient of the magnetic filter is from the very first unable to keep the residual concentration below the desired value. In such a case recourse may be had to two or more magnetic filters connected in series in the oil line — solution which is possible because of the very slight pressure loss of these filters already mentioned. Experience gained until now, however, indicates that for by far the majority of cases the retention coefficient of one filter is more than adequate.

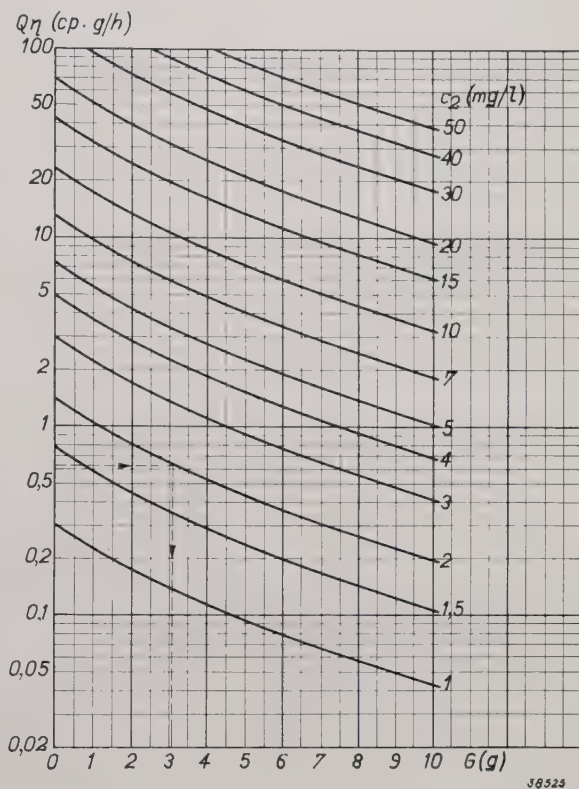


Fig. 12. Graph for use in the practical application of the magnetic filter described. With a given viscosity  $\eta$  of the oil (in cp), a given quantity of new sediment  $Q$  (in g/hr) and a given permissible residual concentration  $c_2$  of the iron (in mg/l), it can be read off on this graph to what degree  $G$  (in grams of iron) the filter may become filled before it must be cleaned.  $G/Q$  is then the corresponding number of working hours. The graph is valid for a rate of flow of 500 l/hr and for iron particles 2 microns in size.



# THE NOISE IN RECEIVING SETS AT VERY HIGH FREQUENCIES

by M. J. O. STRUTT and A. van der ZIEL.

621.396.645.3 : 621.396.822

As a result of the spontaneous current and voltage fluctuations which occur in the input circuit of a receiving set, very weak aerial signals cannot be amplified without interference. The relation between the fluctuation currents and the signal currents in the anode circuit of the first amplifier valve may be taken as a measure of the interference. In this article the factors are investigated which determine the interference with the reception at a given energy of the signal. It is found that at relatively low frequencies (broadcasting frequencies) the fluctuations need not interfere with the reception. At very high frequencies for the case of a triode as first amplifier valve it is found that the interference is determined by the ratio between the noise resistance and the electron-input resistance of the valve. The damping due to the self-induction of the cathode connections has no influence on the ratio between fluctuation current and signal current. Finally the calculations are carried out for pentodes as well. It is shown that the so-called distribution fluctuations may be compensated by introducing a suitable self-induction into the screen grid connections. As to the other fluctuations, the behaviour of a pentode does not differ fundamentally from that of a triode.

When the sensitivity of a receiving set is raised higher and higher, for example by connecting a larger and larger number of stages behind one another, weaker and weaker signals can be received with the desired output amplitude. At the same time, however, it will be found that the reception experiences a larger and larger degree of interference by the familiar noise.

The noise in receiving sets is of fundamental significance in their practical use. *The lower limit of the intensity which an aerial signal must have in order to give satisfactory reception is not determined by the fact that at still lower input voltages the acoustic output signal falls below the threshold value, but by the fact that at still lower input voltage the acoustic output signal is drowned out by the noise.*

The causes of noise in amplifiers have already been discussed several times in this periodical<sup>1)</sup>. As sources of noise were pointed out spontaneous current and voltage fluctuations which may be partially ascribed to thermal causes (Brownian movement) and which are partially connected with the discontinuous corpuscular nature of the electric charge. The weaker the signal which must be dealt with by an amplifier stage, the relatively greater the interference caused by these spontaneous fluctuations. If the ratio between the fluctuation amplitude and the signal amplitude is considered as a measure of the disturbing effect, and if for example the first amplifier stage has a tenfold am-

plification, the spontaneous fluctuations in the second stage have only 10 per cent of the disturbing effect of those in the first stage. If the fluctuation amplitudes are the same in both stages, then due to the fluctuations in the second stage the total fluctuation amplitude increases by a factor  $\sqrt{1^2 + 0.1^2}$ , and therefore becomes only 0.5 per cent greater. The fluctuations in the third and following stages are practically entirely without significance. It is therefore in general the noise in the first and sometimes the second stage which determines the sensitivity of a receiver.

A complete calculation of the spontaneous fluctuations in the first two stages was given some years ago in this periodical<sup>2)</sup>. This treatment is valid only for ordinary and short broadcasting waves. With waves shorter than for instance 10 m, however, these considerations are no longer valid, since at the corresponding frequencies the capacitive and inductive characteristics of the electrode system and of its wiring begin to exert an appreciable influence on the action of the electronic valve. At about the same frequencies the transit time of the electrons between cathode and control grid also begins to affect the action of the valve and especially the noise phenomena<sup>3)</sup>. Both of these effects must be taken into account in the calculation of the sensitivity of an amplifier stage for waves shorter than 10 m. A few extensions of the calculation of the fluctuation phenomena in a receiving set resulting from these effects form the subject of this article.

<sup>1)</sup> M. Ziegler, Philips techn. Rev. 2, 136, 329, 1937.  
C. J. Bakker, Philips techn. Rev., 6, 129, 1941. See also M. J. O. Strutt Spontane spannings- en stroomfluctuaties (ruischen) in elektronenbuizen en aangesloten ketens. (Spontaneous voltage and current fluctuations (noise) in electronic valves and circuits in connection with them), T. Ned. Radio Genootsch. 9, 1, 1941.

<sup>2)</sup> M. Ziegler, Philips techn. Rev., 3, 189, 1938.

<sup>3)</sup> This was pointed out by the authors in the discussion of the modern high-frequency amplifier valves EFF 50 and EF 51, Philips techn. Rev. 5, 172 and 357, 1940.



### Sources of fluctuation in an amplifier circuit

Fig. 1 shows the principle of a high-frequency amplifier stage in which, for the sake of simplicity, it is assumed that the amplifier valve is a triode.  $A$  is an aerial with an internal resistance (radiation resistance)  $R_{ant}$ . The voltages excited in this aerial by the incoming signal are applied, *via* the ideal transformer  $T$ , to an oscillation circuit which is equivalent in tuning to a resistance  $R_{LC}$ . A voltage occurs on this resistance which acts between grid and cathode of the triode, and causes in this way an anode alternating current.

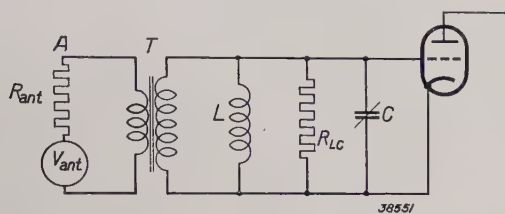


Fig. 1. Principle of the input connections of a receiver.  $A$  aerial with internal resistance (radiation resistance)  $R_{ant}$ ,  $T$  ideal transformer with transformation ratio  $t$ ,  $L$ - $C$  oscillation circuit with resonance resistance  $R_{LC}$ .

In addition to the signal voltage  $V_{ant}$  indicated in the diagram, interference voltages and currents occur at various spots in the circuit due to the fluctuation phenomena mentioned above. Together these lead to certain current fluctuations in the anode circuit of the amplifier valve. *The ratio between the fluctuation current and the signal current in the anode circuit will in the following be considered as a measure of the quality of the reception. The suitability of a valve or of an amplifier connection for the amplification of small signals may be considered greater, the smaller the aerial voltage with which a given quality of reception can be obtained.*

In order to ascertain what factors determine the quality of reception with a given aerial voltage  $V_{ant}$ , we shall consider more closely the different sources of voltage and current fluctuations. Four sources of interference may be distinguished in the connections:

- 1) a disturbance of the aerial signal by other interfering waves in space ( $v_{ant}$ ),
- 2) an interference voltage  $v_c$  in series with the resonance resistance of the oscillation circuit,
- 3) an interference current  $i_k$  in the anode circuit due to the spontaneous fluctuations in the number of electrons and in the velocity at which they are emitted from the hot cathode of the valve,
- 4) a control grid current  $i_g$  correlated with the above which results from the fact that the elec-

trons which are in the space between the electrodes cause the appearance of an induction charge on the control grid which alternates in the rhythm of the anode current fluctuations.

We shall not go into details about these sources, but shall confine ourselves to giving the formulae with which the fluctuation voltages and currents of fig. 1 can be calculated. For a detailed derivation of these formulae we refer to the article of C. J. Bakker cited in footnote <sup>1</sup>).

#### 1) Interfering waves in space

The interference voltage on the aerial which occurs within a frequency region  $\Delta f$  due to interfering waves in space is given by:

$$\overline{v_{ant}^2} = a \cdot 4 k T R_{ant} \Delta f, \dots (1)$$

where  $k = 1.38 \times 10^{-23}$  W sec/degree is the Boltzmann constant, while  $T$  is room temperature in  $^{\circ}\text{K}$  and  $a$  is a numerical factor lying between 1 and  $10^4$ .

#### 2) Interference voltage in series with the oscillation circuit.

For the fluctuation voltage in series with the resonance resistance of the oscillation circuit the following relation is valid in a narrow frequency region close to the resonance frequency:

$$\overline{v_c^2} = 4 k T R_{LC} \Delta f, \dots (2)$$

If  $\Delta f$  is not a very narrow frequency region,  $R_{LC}$  must be defined more precisely, namely as the average value of the real part of the circuit impedance for the frequencies in question. In practice, however, this will not usually involve a very great difference.

#### 3) Spontaneous current fluctuations in the anode circuit of the triode

The spontaneous current fluctuations in the anode circuit of a triode may be described by the formula:

$$\overline{i_k^2} = F^2 \cdot 2 e I_k \Delta f, \dots (3a)$$

where  $e = 1.6 \times 10^{-19}$  coulomb represents the charge on an electron.  $I_k$  is the current which leaves the cathode and  $F$  a factor, the so-called noise factor.

For negative control voltages (starting current region) and for control voltages which have such

<sup>4</sup>) The exact value of  $a$  depends upon the directional characteristic of the aerial. The radiation does not come equally from all parts of the sky, but mainly from the direction of the milky way. If an aerial with a sharp directional effect is used and directed toward the milky way, values of  $a$  in the neighbourhood of 30 are found. See in this connection the article referred to in footnote <sup>1</sup>) by C. J. Bakker in which the literature is given.



a large positive value that saturation occurs,  $F=1$ . In the practical working range of the amplifier valve in which the anode current is limited by space charge,  $F$  is much smaller than unity. The behaviour of  $F$  in this region is such that the fluctuations can be represented in approximation by the relation:

$$\overline{i_k^2} = 4 k T_k S \Delta f \dots \dots (3b)$$

where  $T_k$  is the cathode temperature and  $S$  the slope of the triode <sup>5)</sup>.

Another comprehensive method of representing the current fluctuations  $i_k$  is the introduction of the concept of noise resistance. The noise resistance  $R_r$  of a valve is defined by the relation:

$$\overline{i_k^2} = 4 k T R_r S^2 \Delta f \dots \dots (3c)$$

where  $T$  is room temperature (290 °K).  $R_r$  is the resistance which when connected between cathode and control grid of an electronic valve causes by its thermal voltage fluctuations at room temperature just as large current fluctuations as actually occur spontaneously. From equations (3b) and (3c)  $R_r$  can easily be calculated. For a cathode temperature of 770° C (1 040 °K) we find:

$$R_r \approx 4/S \dots \dots (4)$$

#### 4) Spontaneous current fluctuations in the control grid circuit of the triode

The current fluctuations in the control grid circuit are induced by alternations of the electric charge which is situated in the space between the electrodes. Their behaviour is the same as that of the anode current fluctuations  $i_k$ , but there is a phase shift of 90° with respect to the latter. The intensity of the current fluctuations may be expressed in the form of the input damping  $1/R_e$  which occurs due to the space charge between cathode and control grid. If the space charge between control grid and anode may be neglected, the following relation holds between the current fluctuations and the electron input damping:

$$\overline{i_g^2} = 1,43 \frac{4 k T_k}{R_e} \Delta f \dots \dots (5)$$

If the space charge between control grid and anode

<sup>5)</sup> In the article referred to in footnote <sup>1)</sup> by C. J. Bakker the following relation was derived for a diode:

$$\overline{i_k^2} = 0,64 \cdot 4 k T_k S \Delta f.$$

At the same time it was noted that for a triode a similar expression is valid, except that for  $S$  a value must be taken which is 1.5 to 2 times as large as the measured slope. With the normal amplifier valves this factor is found to be about 1.6, so that the factor 0.64 is thereby increased to about 1.

is important, instead of 1.43 we have a slightly smaller factor, 1 for example.

The mean squares of the sources of fluctuation  $v_{ant}$ ,  $v_C$ ,  $i_k$  and  $i_g$  are determined by the equations (1, 2, 3c and 5). Together with the signal voltage  $V_{ant}$  these data are sufficient to furnish a complete picture of the behaviour of the circuit given in fig. 1.

#### The ratio between fluctuation current and signal current in the anode circuit of the first amplifier valve

After having discussed in the foregoing the four sources of fluctuation of a high-frequency amplifier stage, we shall now study the current fluctuations which are caused in the anode circuit of an amplifier valve in the connections of fig. 1. Since with the help of the same diagram we can also calculate the amplitude of the signal current in the anode circuit, the ratio between fluctuation amplitude and signal amplitude also follows, and in the foregoing we have considered this ratio as a measure of the quality of the reception.

In order to simplify the diagram we omit the ideal transformer  $T$  and replace the aerial resistance as well as the aerial voltages (fluctuation voltages and signal voltage) by their transformed values. If the transformation ratio of the transformer (number of secondary to number of primary windings) is equal to  $t$ , the voltages must be multiplied by a factor  $t$  and the aerial resistance by a factor  $t^2$ . In fig. 2 the diagram so obtained is shown; the electron input resistance  $R_e$  has also been taken into account. The fluctuation voltages  $v_{ant}$  and  $v_C$  and the fluctuation current  $i_g$  cause definite variations in the control grid voltage, which when multiplied by  $S$  give the required fluctuations of the anode current. To this must still be added the spontaneous anode current fluctuations  $i_k$ , i.e. the accidental variations of the anode current which occur due to fluctuations in the mechanism of thermal emission at a constant grid voltage.

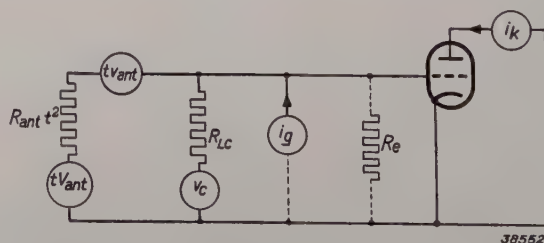


Fig. 2. Equivalent diagram of the input given in fig. 1. The transformer with the aerial connected to it is replaced by an aerial with transformed voltages and transformed radiation resistance. In addition to the aerial voltage  $V_{ant}$ , there are also the fluctuation voltages  $v_{ant}$  and  $v_C$  and the fluctuation currents  $i_g$  and  $i_k$ .



In the performance of the calculations it will be found useful to make repeated use of the following theorem (see fig. 3). A source of voltage  $V$  with an internal resistance  $R$  in series with it can be replaced

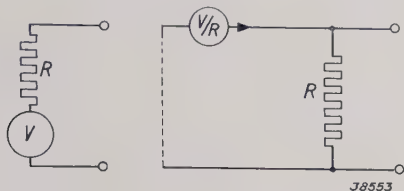


Fig. 3. A source of voltage with an EMF  $V$  and an internal resistance  $R$  corresponds in action to a source of current furnishing a current  $V/R$  to which a resistance  $R$  is connected in parallel. The internal resistance of the source of current (dotted line) must be imagined to be infinitely great.

by a source of current of the strength  $I = V/R$  in parallel with a resistance  $R$ . When this theorem is used it is clear that for example the aerial voltage in fig. 2 can be replaced by a source of current of strength  $V_{ant}/(R_{ant}t)$ , whose current flows through the resistances  $R_{ant}t^2$ ,  $R_{LC}$  and  $R_e$  connected in parallel. If we indicate the resistance value of this system of three resistances connected in parallel by the symbol  $[R_{ant}t^2|R_{LC}|R_e]$ , we find for the control grid A.C. voltage excited by the aerial signal the following:

$$V_g = \{V_{ant}/(R_{ant}t)\} [R_{ant}t^2|R_{LC}|R_e] \quad (6)$$

In a similar way it may be calculated that the voltage sources  $v_{ant}$  and  $v_c$  and the current source  $i_g$  give rise to the following fluctuation voltages on the control grid:

aerial fluctuations:

$$\delta_1 V_g = \{v_{ant}/(R_{ant}t)\} [R_{ant}t^2|R_{LC}|R_e]; \quad (7a)$$

circuit fluctuations:

$$\delta_2 V_g = (v_c/R_{LC})[R_{ant}t^2|R_{LC}|R_e]; \quad (7b)$$

grid current fluctuations:

$$\delta_3 V_g = i_g [R_{ant}t^2|R_{LC}|R_e] \quad (7c)$$

The total grid voltage fluctuation is equal to the sum of the three separate contributions. If this sum is multiplied by  $S$ , the anode current fluctuation thereby caused is obtained. This is thus given by

$$\begin{aligned} \delta I_a &= \delta_1 I_a + \delta_2 I_a + \delta_3 I_a \\ &= S \left( \frac{v_{ant}}{R_{ant}t} + \frac{v_c}{R_{LC}} + i_g \right) [R_{ant}t^2|R_{LC}|R_e]. \end{aligned}$$

If we add to these fluctuations of the anode current the fluctuations

$$\delta_4 I_a = i_k, \quad (7d)$$

we obtain the total anode current fluctuations. Finally we may divide the fluctuation current thus obtained by the signal current  $I_a$  which is obtained by multiplying the grid voltage  $V_g$  (equation 6) by  $S$ . In this way we find for the required ratio between fluctuation current and signal current:

$$\begin{aligned} \frac{\delta I_a}{I_a} &= \frac{v_{ant}}{V_{ant}} + \\ &+ \frac{R_{ant}t}{V_{ant}} \left\{ \frac{v_c}{R_{LC}} + i_g + \frac{i_k}{S [R_{ant}t^2|R_{LC}|R_e]} \right\}. \quad (8) \end{aligned}$$

### The influence of the self-induction of the cathode connections at very high frequencies

At very high frequencies the self-inductions and mutual inductions of the connections of the valve and the wires used for the connections may have an important effect on the action of an amplifier stage, so that there may be some doubt whether fig. 2 represents a usable equivalent circuit diagram of the amplifier stage. In particular the self-induction of the cathode connections leads to the fact that the admittance of the valve between cathode and control grid is not given only by the electron input damping  $1/R_e$ , but is several times larger with the normal dimensions of high frequency amplifier valves. If the total damping is written in the form

$$\frac{1}{R_i} = \frac{1}{R_e} + \frac{1}{R_{L_k}}, \quad (9)$$

then for the share of the cathode connections in the damping the following may be calculated<sup>6)</sup>:

$$\frac{1}{R_{L_k}} = \omega^2 L_k C_{kg} S, \quad (10)$$

where  $L_k$  is the self-induction of the cathode connections and  $C_{kg}$  is the capacity between cathode and control grid.

<sup>6)</sup> See Philips techn. Rev., 3, 103, 1938. The derivation is briefly as follows. If  $V$  is the A.C. voltage between the control grid and the end of the cathode pin, and if  $I_a$  is the anode alternating current, the A.C. voltage between control grid and cathode is approximately

$$V_g = V - j\omega L_k I_a$$

and thus the alternating current which flows in the grid circuit as a result of the capacity  $C_{kg}$  between cathode and control grid is

$$I_g = j\omega C_{kg} V_g = j\omega C_{kg} V + \omega^2 L_k C_{kg} I_a.$$

As a first approximation  $I_a = SV$ , where  $S$  represents the slope of the valve. Thus.

$$I_g = (j\omega C_{kg} + \omega^2 L_k C_{kg} S) V.$$

The first term in parenthesis represents a capacitive admittance which can be eliminated by connecting a suitable self-induction in parallel (this is done automatically in tuning the input circuit). The last term then remains and this is just the ohmic admittance  $1/R_{L_k}$ .



With the help of *fig. 4* we shall now study how the signal current and the fluctuation current in the anode circuit of the amplifier valve change due to the self-induction of the cathode connections.

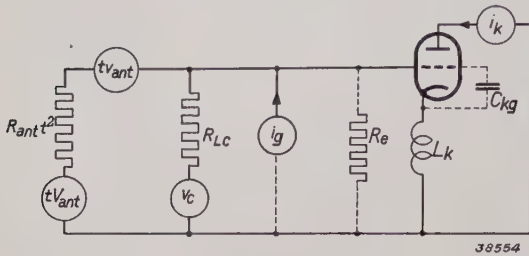


Fig. 4. Extended equivalent circuit diagram of the connections given in *fig. 1*. Besides the elements shown in *fig. 3* the self-induction  $L_k$  of the cathode connections and the capacity  $C_{kg}$  between cathode and control grid have been taken into account.

At first glance it might be supposed that it is simply necessary to replace the input resistance  $[R_{ant} t^2 | R_{LC} | R_e]$  between control grid and cathode by the smaller value  $[R_{ant} t^2 | R_{LC} | R_i]$  everywhere in the formulae. Since  $I_a$  as well as  $\delta_1 I_a$ ,  $\delta_2 I_a$  and  $\delta_3 I_a$  change proportionally with the input resistance, the quotient  $\delta I_a / I_a$  would not be affected by this change, as far as the first three sources of fluctuation are concerned. The fourth source of fluctuation  $\delta_4 I_a$ , however, appears according to (7d) to be independent of the input resistance, so that the relative intensity of these fluctuations would have to increase with decreasing input resistance. Further considerations which will be given below, will, however, show that this conclusion is incorrect; the relative intensity of these last fluctuations also remains quite unchanged. The self-induction of the cathode connections of an amplifier valve thus has no effect on the ratio between fluctuation current and signal current, so that equation (8) need not be corrected for this.

The decrease of  $\delta_4 I_a$  with increasing damping due to the self-induction of the cathode connections may be explained as follows. A spontaneous increase in the anode current causes a change in the cathode potential due to the self-induction of the cathode connections. Due to the capacity between control grid and cathode, this change causes the flow of grid currents, which in turn cause the control grid potential to change, in the sense that it decreases. Hereby the spontaneous increase of the anode current is partially suppressed; the remaining effect is found to be given by the formula:

$$\delta_4 I_a = i_k \frac{[R_{ant} t^2 | R_{LC} | R_i]}{[R_{ant} t^2 | R_{LC} | R_e]}, \dots \quad (11)$$

so that  $\delta_3 I_a$  actually does change proportionally with the input resistance  $[R_{ant} t^2 | R_{LC} | R_i]$ .

In order to confirm equation (11) let us consider the equivalent

circuit given in *fig. 5a*. Due to the anode current fluctuation  $\delta_4 I_a$  a voltage  $j\omega L_k \delta_4 I_a$  occurs over the self-induction  $L_k$ . Since the impedance of the capacity  $C_{kg}$  connected in series with this is still large, even at very high frequencies, compared with that of  $L_k$ , we may consider  $L_k$  in the circuit sketched as a source of voltage without internal resistance. By again applying the theorem of *fig. 3* to the branch  $L_k$ - $C_{kg}$ , we transform

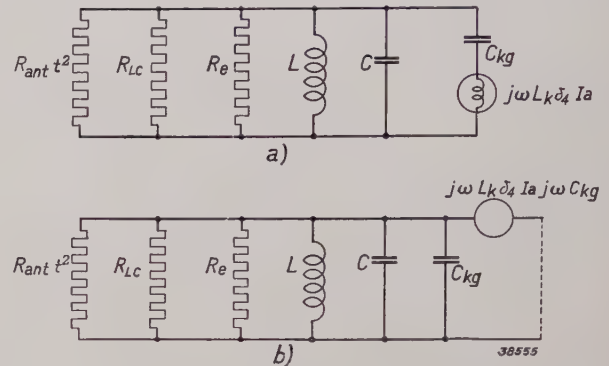


Fig. 5. Diagram showing the reaction of the anode circuit on the control grid circuit *via* the self-induction  $L_k$  and the capacity  $C_{kg}$ . Diagram *b)* is derived from *a)* by applying the theorem of *fig. 3* to the branch  $L_k$ - $C_{kg}$ .

the voltage source into a current source again (see *fig. 5b*). The current from the source is  $j\omega L_k \delta_4 I_a \cdot j\omega C_{kg}$ . The current is divided among the branches  $R_{ant} t^2$ ,  $R_{LC}$ ,  $R_e$ ,  $L$ ,  $C$ ,  $C_{kg}$ . We shall now consider only those components of the current fluctuations with frequencies in the neighbourhood of the resonance frequency. For these the following is valid:

$$\omega^2 L (C + C_{kg}) = 1$$

In the neighbourhood of this frequency the currents through the reactive branches  $L$ ,  $C$ ,  $C_{kg}$  practically cancel each other, so that the whole current from the source flows through the resistance branches. This current therefore causes a grid voltage variation (voltage across  $C_{kg}$ ) of the magnitude:

$$\delta_4 V_g = j\omega L_k \delta_4 I_a j\omega C_{kg} [R_{ant} t^2 | R_{LC} | R_e].$$

The result of this grid voltage variation is an anode current fluctuation  $\delta_4 V_g \cdot S$ . By adding to this the anode current fluctuation  $i_k$  we obtain the total anode current fluctuation  $\delta_4 I_a$ , which is thus given by the relation:

$$\delta_4 I_a = -\omega^2 L_k C_{kg} S [R_{ant} t^2 | R_{LC} | R_e] \delta_4 I_a + i_k$$

This equation is easily solved for  $\delta_4 I_a$  and one obtains

$$\delta_4 I_a = \frac{i_k}{[R_{ant} t^2 | R_{LC} | R_e] \left( \frac{1}{[R_{ant} t^2 | R_{LC} | R_e]} + \omega^2 L_k C_{kg} S \right)}$$

If, finally, we take into account the fact that  $\omega^2 L_k C_{kg} S = 1 / R_{Lk}$  (equation 10), we may simplify the factor in parenthesis in the following way:

$$\frac{1}{[R_{ant} t^2 | R_{LC} | R_e]} + \frac{1}{R_{Lk}} = \frac{1}{[R_{ant} t^2 | R_{LC} | R_i]}$$

and we thus obtain:

$$\delta_4 I_a = i_k \frac{[R_{ant} t^2 | R_{LC} | R_i]}{[R_{ant} t^2 | R_{LC} | R_e]},$$

which confirms equation (11).



### Discussion of the result

Equation (8) gives the ratio between the fluctuation current and the signal current in the anode circuit of the first amplifier valve as a function of the signal voltage and of the various fluctuation voltages and currents. Now the fluctuation voltages and currents themselves are not known; the theoretical formulae which were given at the beginning of this article only give the mean squares of these fluctuations. Therefore we can only use equation (8) when both sides are squared and the mean value of that power is considered. If we work out the third term in braces and consider also that the mean values of the product of two different fluctuations are equal to zero <sup>7)</sup> we obtain:

$$\frac{\overline{\delta I_a^2}}{\overline{I_a^2}} = \frac{\overline{v_{ant}^2}}{\overline{V_{ant}^2}} + \frac{R_{ant}^2 t^2}{\overline{V_{ant}^2}} \left\{ \frac{\overline{v_c^2}}{R_{LC}^2} + \overline{i_g^2} + \frac{\overline{i_k^2}}{S^2} \left( \frac{1}{R_{ant} t^2} + \frac{1}{R_{LC}} + \frac{1}{R_e} \right)^2 \right\} \cdot \cdot \cdot \cdot \cdot \quad (12)$$

The values  $\overline{v_{ant}^2}$ ,  $\overline{v_c^2}$ ,  $\overline{i_g^2}$  and  $\overline{i_k^2}$  have already been given in the beginning of this article by the formulae (1), (2), 3c) and (5). With these formulae we obtain:

$$\frac{\overline{\delta I_a^2}}{\overline{I_a^2}} = 4 k T \Delta f \frac{R_{ant}^2 t^2}{\overline{V_{ant}^2}} \left[ \frac{\alpha}{R_{ant} t^2} + \frac{1}{R_{LC}} + \frac{5,6}{R_e} + R_r \left( \frac{1}{R_{ant} t^2} + \frac{1}{R_{LC}} + \frac{1}{R_e} \right)^2 \right] \cdot \cdot \cdot \cdot \quad (13)$$

In this expression the relation between fluctuation current and signal current is represented as a function of aerial properties, circuit properties and valve properties.

A quantity which has not been discussed until now is the transformation ratio  $t$ . There are various points of view to be considered in the choice of this ratio.

In the case of the ordinary broadcasting receiving sets it is considered particularly important that the resistance and the capacity of the aerial should not cause too great a detuning of the input circuit, so that a set with a calibrated scale can be connected to different aerials. For this reason the transformation ratio  $t$  is chosen very large. It may be seen from equation (6) that the signal voltage on the control grid approaches zero for very large as well as for very small values of the transformation ratio; increasing  $t$  beyond a certain limit will therefore be impossible without a loss of selectivity.

If it is desired to be able to receive very weak signals, it is obvious that  $t$  must be chosen so that the sensitivity is as great as possible. The condition for this, derived from equation (6) is the following:

$$1/t^2 = R_{ant} \left( \frac{1}{R_{LC}} + \frac{1}{R_i} \right) \cdot \cdot \cdot \cdot \cdot \quad (14a)$$

With this choice of  $t$  the largest signal is obtained at the grid of the amplifier valve. This does not, however, immediately mean that the quality of the reception with this choice of  $t$  will be the best. As mentioned in the foregoing the requirement of the best possible reception leads to the condition that the signal amplitude  $I_a$  should not be as large as possible in absolute value, but in relation to the fluctuation amplitude  $\delta I_a$ . If from equation (13) the value of  $t$  is calculated for which this condition is fulfilled, one finds that

$$1/t^2 = R_{ant} \sqrt{\left( \frac{1}{R_{LC}} + \frac{1}{R_e} \right)^2 + \frac{1}{R_r} \left( \frac{1}{R_{LC}} + \frac{5,6}{R_e} \right)} \cdot \cdot \cdot \cdot \cdot \quad (14b)$$

*i.e.* a relation which will often furnish a considerably smaller value of the transformation ratio. If  $t$  is chosen according to equation (14) the following relation is obtained for the ratio between fluctuation current and aerial current:

$$\frac{\overline{\delta I_a^2}}{\overline{I_a^2}} = \frac{4 k T \Delta f}{\overline{V_{ant}^2} / 2 R_{ant}} \left[ \frac{\alpha}{2} + R_r \left( \frac{1}{R_{LC}} + \frac{1}{R_e} \right) + \sqrt{\frac{R_r}{R_{LC}} + 5,6 \frac{R_r}{R_e} + R_r^2 \left( \frac{1}{R_{LC}} + \frac{1}{R_e} \right)^2} \right] \cdot \cdot \cdot \cdot \quad (15)$$

For a given signal energy and given values  $R_r$ ,  $R_{LC}$  and  $R_e$  this formula gives the optimum ratio between fluctuation current and signal current. We shall now consider two limiting cases, namely the case of normal broadcasting frequencies and the case of very high frequencies.

At *broadcasting frequencies*  $R_e$  amounts to many megohms for ordinary amplifier valves, while  $R_r$  is in

<sup>7)</sup> This is always the case when two chance fluctuations are statistically independent of each other. In the case of the fluctuations  $i_g$  and  $i_k$  this condition is not fulfilled. Never-

theless the average of the product it disappears, since the two fluctuations have a relative phase difference of practically 90°.



general only several hundred ohms (see equation (4)). The contribution of the electron input damping to the noise may then be entirely neglected.

If we neglect the cosmic noise, which does not actually represent any lack of perfection of the receiver, but a lack of perfection in the incoming signal, the  $\alpha$  between the brackets disappears. The ratio between fluctuation current and signal current at low frequencies is then almost determined by  $R_r/R_{LC}$ , and we see that the receiver can be made suitable for the reception of extremely weak signals by choosing the circuit resistance  $R_{LC}$  sufficiently large. As an objection to this it is often put forward that the circuit resistance should not exceed a certain value, since otherwise an undesired high selectivity is obtained. This objection is, however, unjustified: the selectivity is not determined by the circuit resistance  $R_{LC}$ , but by the total input resistance  $[R_{ant}t^2/R_{LC}|R_i]$  of the connections. Now among other things  $R_i$  contains the damping due to the self-induction of the  $e_2$  cathode connections. As we have seen, this contribution to the damping has no influence on the ratio of fluctuation current to signal current; we may therefore reduce an undesired high value of the total input resistance to the desired value with the help of this damping term. In this way we can obtain a receiving circuit which amplifies an extremely weak input signal almost free of noise at not too high frequencies.

At very high frequencies ( $<10$  m), in addition to the circuit damping the electron input damping is also very important. With waves of the order of magnitude of 50 cm the electron input damping  $R_e$  amounts to only a few hundred ohms even for the best valves now in use, while the resistance in parallel  $R_{LC}$  of the oscillation circuit can be made almost as large as with broadcasting frequencies by the use of cavity resonators and the like. The ratio between fluctuation current and signal current is then determined according to equation (15) exclusively by  $R_r/R_e$ , thus by the ratio between noise resistance and electron input resistance of the valve. *The quotient  $R_r/R_e$  thus determines the suitability of an amplifier valve for the amplification of very weak signals of high frequency.* Use has already been made in this periodical of this result, namely in the estimation of the properties of the valves EFF 50 and EF 51 <sup>8)</sup>.

### The compensation of distribution fluctuations in pentodes <sup>8)</sup>

In the foregoing we have assumed that a triode was used as amplifier valve. It is not, however, difficult to carry out the same calculations for valves with more grids. Since a pentode is usually used as high-frequency amplifier valve, we shall examine the fluctuation phenomena occurring in this case more closely. In a pentode the same fluctuation phenomena are present as in a triode, but in addition so-called distribution fluctuations occur due to a changing current distribution between screen grid and anode.

If the self-induction of the cathode connections is neglected, it is found that the influence of the distribution fluctuations can be taken into account by an increase of the noise resistance  $R_r$  (for instance by a factor 2), and for the rest, the formulae derived above for the triode may be retained. If, however, the self-induction of the cathode connections has a significant effect, less pleasing results are obtained. The signal currents and the fluctuation currents calculated until now are weakened in the same ratio by this self-induction. The distribution fluctuations, however, which do not pass through this self-induction, remain unchanged, and will

therefore cause a relatively greater interference <sup>9)</sup>.

A fundamentally simple manner of overcoming this objection is indicated in fig. 6. The screen grid

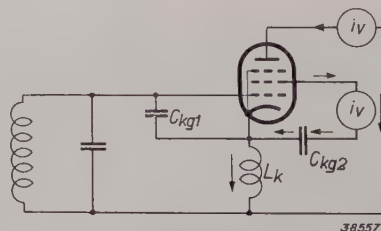


Fig. 6. By introducing a large enough capacity  $C_{kg2}$  between screen grid and cathode inside the valve, the distribution fluctuations  $i_v$  can be made to pass through the self-induction  $L_k$  of the cathode connections.

is connected to the cathode inside the tube via a sufficiently large capacity  $C_{kg2}$ . The screen grid

<sup>8)</sup> See the articles referred to in footnote <sup>3)</sup>, as well as M. J. O. Strutt and A. van der Ziel, *Physica* **8**, 1 and 424, 1941.

<sup>9)</sup> This phenomenon is more objectionable the more the signal is damped by the self-induction of the cathode connections. From this point of view the valves EFF 50 and EF 41, which have recently been described in this periodical, are very satisfactory. In the case of the valve EFF 50 the self induction of the cathode connections, thanks to the application of the push-pull principle, furnishes only a very small contribution to the input damping, while in the case of the valve EF 50, by the use of a second cathode connection, it has even been made possible that the self induction of the cathode connection does not damp but eliminates damping.



current then also flows through the cathode connections as indicated by arrows in the figure. It may be seen that now, as far as the fluctuations are concerned, the screen grid simply works as a second cathode connected in parallel with the first, so that the fluctuations  $i_v$  can be added directly to the fluctuations  $i_k$ . The result is an increase in the noise resistance  $R_r$ , but for the rest the formulae derived in the foregoing hold exactly with this altered noise resistance.

The connections indicated in fig. 6, however, do not yet represent the best method of using the pentode. The ratio between fluctuation current and signal current in the anode connections can be made still smaller if a self-induction is included not only in the cathode connections but also in the screen grid connections. We then obtain the connections shown in fig. 7. If for this scheme we exam-

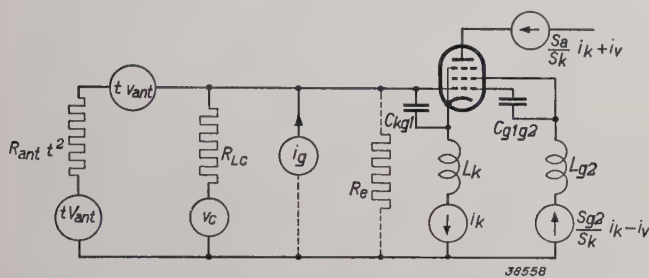


Fig. 7. Substitute diagram for an input circuit with pentode. Besides the self induction  $L_k$  of the cathode connections and the capacity  $C_{kg}$  between cathode and control grid, the self induction  $L_{g2}$  of the screen grid connections and the capacity  $C_{g1g2}$  between control grid and screen grid affect the properties of the circuit.

ine the action of the signal voltage  $V_{ant}$  and of the fluctuation voltages and currents  $v_{ant}$ ,  $v_c$ ,  $i_g$  and  $i_k$ , we find that the currents which are excited by these sources of voltage and current in the anode circuit of a pentode can be described by exactly the same formulae as in the case of the triode, except that for  $R_i$  a slightly different value must be taken, since this input resistance is found to depend upon  $L_{g2}$  as well as  $L_k$ .

We may therefore say that the properties of a

pentode with relation to the aerial signal and the sources of fluctuation  $v_{ant}$ ,  $v_c$ ,  $i_g$  and  $i_k$  do not differ appreciably from those of a triode. The great difference lies, however, in the fact that in the pentode there is a fifth source of fluctuation, namely the above-mentioned distribution fluctuations  $i_v$ .

What is the result of this on the fluctuations in the anode circuit? When there is no self-induction in the screen grid connections the distribution fluctuations must simply be added to the other fluctuations of the anode current. If, however, there is a self-induction in the screen grid connections, an attenuation of the distribution fluctuations occurs in the anode circuit due to a mechanism entirely analogous to that which is responsible for the attenuation of the cathode current fluctuations. We shall not therefore go more deeply into the calculation, but state only that a distribution fluctuation  $i_v$  in the screen grid circuit reacts on the control grid voltage and thereby leads to an anode current fluctuation:

$$i_v' = -i_v \omega^2 L_{g2} C_{g1g2} S [R_{ant} t^2 |R_{LC}| R_i]^{10}.$$

The total distribution fluctuation in the anode circuit is

$$\begin{aligned} \delta_5 I_a &= i_v + i_v' = \\ &= i_v (1 - \omega^2 L_{g2} C_{g1g2} S [R_{ant} t^2 |R_{LC}| R_i]). \end{aligned}$$

It is therefore clear that the distribution fluctuations in the anode circuit can be made equal to zero by choosing a suitable value for the self-induction  $L_{g2}$  of the screen grid connection.

If the distribution fluctuations have been compensated, the pentode behaves exactly like a triode as far as the fluctuations are concerned.

<sup>10)</sup> This formula is entirely analogous to the formula for the fluctuations in the anode circuit which are excited by the cathode current fluctuations  $i_k$ . The latter may be written in the form

$$\delta_4 I_a - i_k = i_k' = -i_k \omega^2 L_k C_{kg} S [R_{ant} t^2 |R_{LC}| R_i]$$

as may easily be deduced from the formulae given for  $\delta_3 I_a$  in the foregoing.



## A METHOD FOR THE CONTROL OF COLOUR DEVIATIONS

by J. J. WENT and P. KOOLE.

535.6.08

In the case of coloured products it is often necessary to be able to reproduce a given colour exactly at some later time. For practical purposes it is very desirable to be able to ascertain objectively whether this requirement is satisfied or to be able to indicate objectively fixed tolerances for this condition. A simple method of doing this has been worked out and is described in this article. The arrangement is intended particularly for controlling the colours of lacquers. The "colour" of a lacquer is measured by the reflection coefficient for four separate wave lengths. The practical application of the method is illustrated by means of an example.

Wherever coloured substances or articles are made the question may arise of whether the colour obtained is "correct", i.e. whether it corresponds exactly to what was desired. This question may be of particular importance when it is necessary at some later date to reproduce a given colour exactly, which is often the case in the dyeing of textiles, the lacquering of surfaces and the like. The system ordinarily used in the past for the control of colour consisted in keeping a dyed or lacquered sample, with which the coloured object could be compared visually. This method was usually modified to the extent that a sample of the pigment or lacquer itself was kept and at the moment when the comparison took place a new test object was dyed or lacquered with the old sample.

There are two objections to this method. In the first place uncontrollable colour changes may occur during the time the sample is kept. In the second place visual comparison involves the fact that only a subjective judgement can be obtained about the presence of any colour deviations and whether or not they may be permissible. Difficulties often arose between manufacturers and consumers due to this lack of objectively fixed and controllable colour tolerances.

Particularly for the second example mentioned above, lacquered surfaces, a simple method has been worked out in this laboratory which permits the determination of the colour by an objective measurement. This method will be described.

### Principle of the method

The colour impression of a surface, for instance of a lacquered plate, is due to the fact that light of different wave lengths is reflected in different intensities. From this it follows directly that the colour impression will depend, among other factors, upon the spectral distribution of the light with which the surface is irradiated. If this spectral distribution is known, and if the reflection coefficient  $R$  of the plate is known for all visible wave lengths  $\lambda$ ,

the composition and thus also the colour of the reflected light can be calculated<sup>1)</sup>.

In our case it is not actually a question of the colour itself, but of similarity of colour between two surfaces, no matter by what light source they are both irradiated. In order to ensure this similarity the function  $R(\lambda)$  will clearly have to be the same for the two surfaces. The measurement of the "colour" of lacquers which we are here considering thus reduces to the determination of the function  $E(\lambda)^2$ .

Two surfaces with the same value of  $R(\lambda)$  may not actually have the same appearance. Besides the dependence on wave length the dependence on angle of the reflective coefficient will play a part. If a glossy and a dull surface are compared, the former will reflect especially strongly in those directions in which the angle of observation  $\alpha$  is equal to the angle of incidence  $\beta$  of the light, while for a dull surface there is a more uniform distribution over all angles  $\alpha$ . If therefore absolute similarity of two surfaces is required, not only must the functions  $R(\lambda)$  correspond for a given angle of incidence and observation, but in general the functions  $R(\lambda, \alpha, \beta)$  must correspond.

The variation of the function  $R(\lambda, \alpha, \beta)$  with the angles  $\alpha$  and  $\beta$  depends not only on the kind of lacquer, but to a very high degree upon the nature of the surface to which the lacquer is applied, as well as upon the way in which it is applied (sprayed, brushed, dried quickly or slowly, etc.). In order to eliminate these factors the lacquer surface to be

<sup>1)</sup> Besides the colour impression, the reflection coefficients also determine the brightness impression which is just as important in practice. For the first factor it is mainly the shape of the curve  $R(\lambda)$ , which is decisive, for the second, the size of  $R(\lambda)$ . For the sake of simplicity we speak here only of the "colour"; at the end of the article the two concepts will be separated again.

<sup>2)</sup> The determination of a colour by a point in the colour triangle, as has been described and applied in this periodical (e.g. P. J. Bouma, Philips techn. Rev. **1**, 282, 1936 and recently by A. A. Kruithof, Philips techn. Rev. **6**, 65, 1941) would not be sufficient here, because a given surface may have very different colour points according to the nature of the illuminating source.



investigated was applied to a glass plate and the reflection coefficient of the lacquer surface against the glass was measured, see *fig. 1*. Because of the good reproducibility of this kind of surface it may in this case be assumed that with the same kind of lacquers the  $R(\lambda, a, \beta)$  functions will have the same character as far as their dependence on  $a$  and  $\beta$  is concerned, and will only differ in absolute size

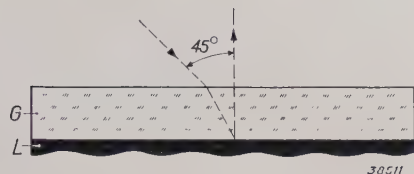


Fig. 1. A layer of lacquer  $L$  is applied to a glass plate  $G$ . The reflection of the lacquer surface against the glass is investigated for the angles of incidence and reflection indicated. Taking the refraction in the glass into account, the light must be incident on the plate at an angle of about  $45^\circ$ .

(i.e. in the total reflection coefficient). It is therefore possible in measuring  $R$  to confine oneself to a single angle of incidence and a single angle of observation. For the former an angle of  $30^\circ$  was chosen, while the observation took place approximately normal to the surface.

The use of glass test plates has also a second advantage. When a layer of lacquer does not "cover" because it has not been made thick enough, the reflectivity of the underlayer will also play a part, and the value measured will not be a true property of the lacquer alone. It cannot be seen directly whether or not a layer of lacquer covers entirely unless it is applied in the manner described to a transparent underlayer, since when this is done a covering layer of lacquer will transmit practically no light.

Although it is now only necessary to measure the reflection coefficients in a single definite arrangement, the determination of the complete function  $R(\lambda)$  would still require a large number of measurements. As a further simplification, therefore, the reflection coefficient is determined for only four definite wave lengths which are distributed as favourably as possible throughout the whole visible region of the spectrum.

There may of course be dangers connected with this simplification. If one is concerned with the two reflection curves drawn in *fig. 2* for example and if the reflection coefficient is measured at exactly the four wave lengths  $\lambda_1$ - $\lambda_4$  where the curves intersect each other, the surfaces will be judged to be identical, while they will certainly not give the same colour impression under all conditions of illu-

mination<sup>3</sup>). In practice, however, this objection is not so serious. In the first place the reflection curves of the most common pigments have a smooth flowing shape like curve 1 in *fig. 2*, and only a few

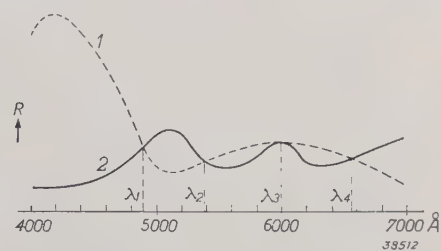


Fig. 2. A case may occur in which the curves  $R(\lambda)$  of two different surfaces cut or touch each other at four points (wave lengths  $\lambda_1$ - $\lambda_4$ ).

pigments such as occur in oil paintings are observed to have several maxima and minima (curve 2 in *fig. 2*). The fact that two reflection curves under examination should have four points of intersection is therefore already very improbable, and that these four points of intersection should correspond exactly to the four wave lengths measured would be extremely improbable. In the second place the problem here is the comparison of approximately similar surfaces which will in principle contain the same pigments, and which therefore must exhibit qualitatively similar behaviour in their reflection curves, see *fig. 3*.

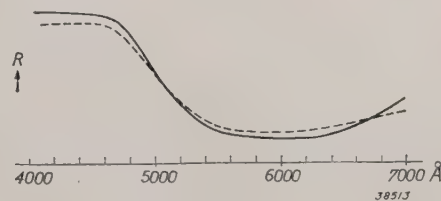


Fig. 3. Ordinary kinds of paints and lacquers have fairly smooth  $R(\lambda)$  curves. The curves of two lacquers to be compared will not in general have more than two, at the most three, common points, as shown in the figure.

### Description of the measuring arrangement

For the measurement of the reflection coefficient according to the method outlined four kinds of monochromatic light distributed over the whole spectrum are required. The following were chosen: in the blue, green and yellow the mercury lines 4 358, 5 461 and 5 780 Å, respectively, while for

<sup>3</sup>) The danger becomes less when, instead of taking four separate wave lengths, the whole spectrum is divided into four connecting wave length regions and the measurement is carried out in these regions. Such an apparatus which works with eight instead of four "blocks" is the block photometer described in this periodical (P. M. v. Alphen, Philips techn. Rev. 4, 66, 1939). The arrangement would however, become much more complicated and expensive if this principle were applied here.



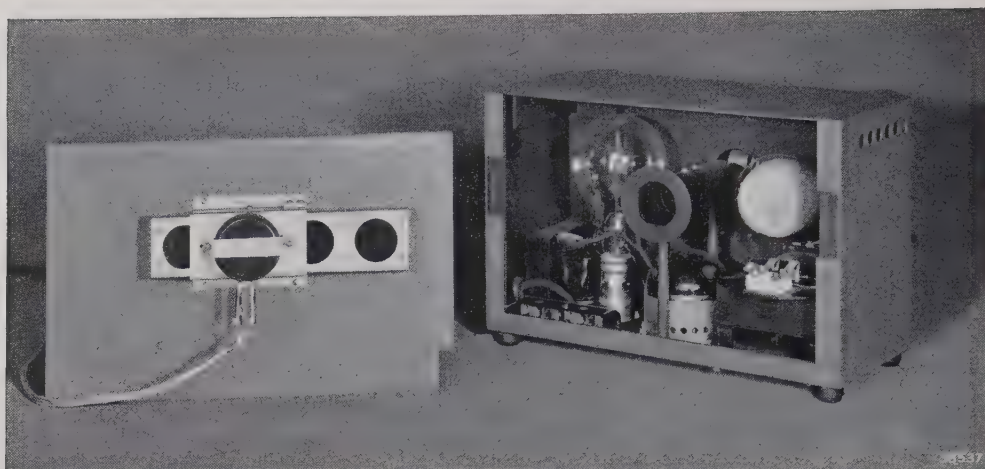


Fig. 4. The cabinet with the measuring arrangement with its front wall removed and placed beside it on the left. To the left inside the cabinet may be seen the mercury lamp, to the right the ordinary electric lamp (with half of the bulb silvered), in addition the series apparatus needed for the mercury lamp. In the middle is a tube containing a lens and below it a fan. On the outside of the front wall is a slide, with four filters and in the middle a selenium photocell.

the red, in which it is difficult to obtain a monochromatic radiation of sufficient intensity, a spectral band was used which cuts off sharply at one edge at  $6\,000\text{ \AA}$  and falls off at the other edge at about  $7\,000\text{ \AA}$ . As sources of light an ordinary high-pressure mercury lamp (HP 300 Dlm) and for the red an ordinary gas-filled electric lamp of 500 W were used. In the photograph *fig. 4* the two lamps may be seen to the right and left in the cabinet whose front wall has been removed. The plate to be investigated is pressed by means of a clamp against a circular opening in the rear wall of the cabinet (see *fig. 5*). The lamps are so placed that the plate is illuminated at an angle of about  $45^\circ$ , and due to the refraction in the glass the angle of incidence on the laquer surface amounts to about  $30^\circ$ . The part of the light reflected in about the

direction of the normal to the surface is concentrated by a lens on a photoelement (selenium blocking-layer cell) which is set up against the front wall (see *fig. 6*). A tube shields the photocell from other directions. With the help of a slide between lens and cell different filter combinations can be brought into the path of the light. These combinations are so chosen that in each case only one of the three desired spectral lines or the spectral band in the red is transmitted.

Since, after reflection at the test plate and filtering, only a fraction of the original light flux reaches the photocell, in order to obtain easily measurable photocurrents the original light flux must be adequate. On the other hand the light flux must not be too large because the method of measurement to be described in the following makes it

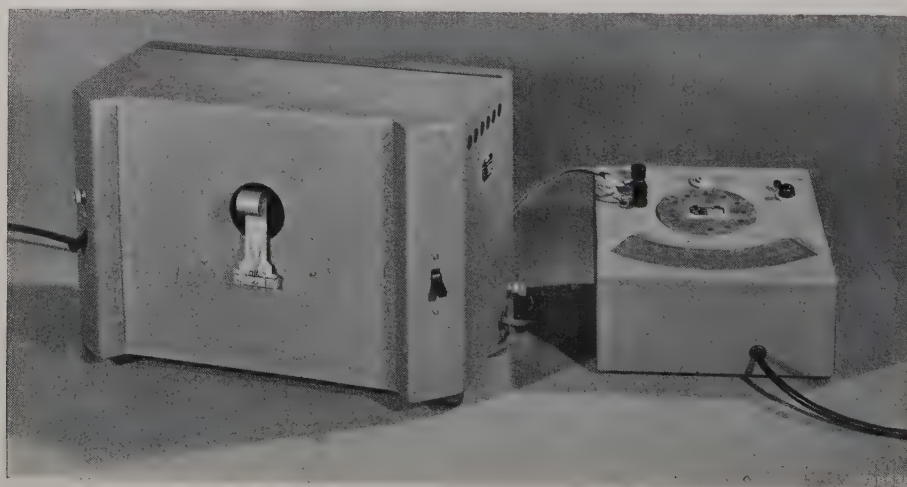


Fig. 5. Rear view of the cabinet. The circular opening against which the test plate can be pressed is visible. With the two switches to the left and right the mercury lamp and the electric lamp, respectively, can be switched on. To the right a microammeter.



desirable to have a linear relation between the photocurrent and the light flux incident on the cell. With the lamps mentioned above this linearity is ensured for all spectral regions and reflectivities of practical importance, while the photocurrents are still large enough to be measured directly with a microammeter, *i.e.* without any amplifier.

In order to avoid the development of too much heat inside the cabinet while the lamps are burning, a small fan has been built in. This may be seen in fig. 4 below the tube. It is switched on at the same time as the lamps.

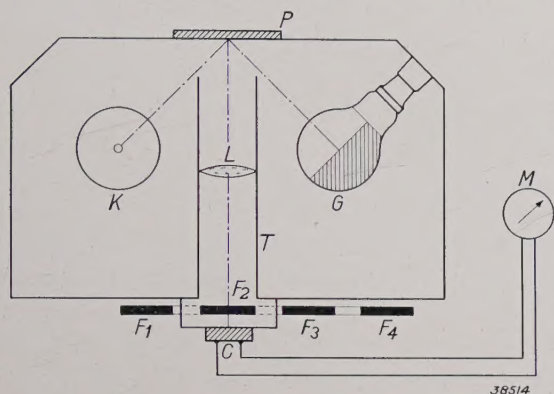


Fig. 6. Path of the rays in the measuring arrangement. *K* mercury lamp, *G* electric lamp, *P* test plate, *L* lens, *T* tube, *F*<sub>1</sub>-*F*<sub>3</sub> filters, *C* selenium photocell, *M* microammeter.

### Performance of the measurements

In order to determine the absolute value of the reflection coefficients, the light intensity of the lamps, the transmission of the filters, the sensitivity of the cell, etc. would have to be taken into account as functions of the wave length. Since, however, it is here only a question of the comparison of reflection coefficients, this is unnecessary. It is enough to compare the photo-currents obtained with the test surface or with a given standard surface. If the relation between photocurrent and light flux incident on the cell is linear, the ratio between the meter readings obtained with two surfaces is equal to the ratio between the two reflection coefficients. The required reflection coefficient is thus found in this way in per cent of the reflection coefficient of the standard surface.

In the first instance it is a matter of indifference what is used as standard surface, except that it must be satisfactorily reproducible as far as its  $R(\lambda, \alpha, \beta)$  function is concerned, and it must remain constant indefinitely. In order to compare results obtained at different places with different apparatus a standard is in any case needed which satisfies the first requirement (primary standard). A very suitable surface for this purpose is that

obtained by burning magnesium and allowing the magnesium oxide formed to be deposited on a plate. When sufficiently thick this surface reflects 95 per cent of the incident light very nearly according to Lambert's law (ideal diffuse reflection). In order to judge whether the layer is thick enough a mirror is used as foundation: it is only necessary to continue the deposition of magnesium oxide until no specular reflection can be observed.

The surface so obtained is easily reproducible, but at the same time very delicate. For the measurements proper, therefore, a more durable secondary standard will be used, for instance a frosted milk glass plate which can be calibrated with the primary standard in order to relate the results with different apparatus to each other. The numerical values of the relative reflection coefficients mentioned in the example below have been referred in this way to the magnesium-oxide standard.

### Example of application

By the measurement of the relative reflection coefficients for four reference wave lengths we can now ascertain whether or not two lacquer surfaces are similar. If they are not similar, and there will practically always be small differences, the question immediately arises as to how large these differences may be before this actually becomes observable as a changed impression of colour or brightness. A definite (objective) answer to this question cannot be given, since only a subjective judgement is possible in the decision whether two surfaces differ visibly or not. If, however, two given surfaces have once been qualified as sufficiently alike, by common agreement between producer and consumer, and others as too different, then with the help of the arrangement here described the differences in reflection coefficients which correspond to the one case and to the other can be determined, and thereby the agreed colour tolerances are fixed for later use, and are always available.

As an example we shall discuss the investigation of a certain light grey lacquer which is used for lacquering transmitter panels and the like. This lacquer is prepared by mixing four basic pigments, namely dull white, dull black, dull red-brown and blue. The relative reflection coefficients of the original sample of the lacquer amount (in per cent of the reflection of magnesium oxide) to: for the red reference wave length  $r = 26.5$  per cent, for the yellow  $g = 30.5$  per cent, for the green  $gr = 32.5$  per cent and for the blue  $b = 36.1$  per cent.

An investigation was made of the way in which the reflection coefficients for the four reference wave



lengths change when definite additional amounts of one or more of the basic pigments are added to the light grey lacquer. The difference found between the per cent values of the original and the new reflection coefficient ( $\Delta R$ ) is plotted in fig. 7. for the four reference wave lengths ( $r, g, gr, b$ ) against the per cents of extra pigment added. In fig. 7a white pigment was added and the reflection increased; in figs. 7b, c d, black, blue and red-brown pigment, respectively, was added, and the reflection decreased. In fig. 7e black and white were finally added at the same time (in a ratio of 4 to 1).

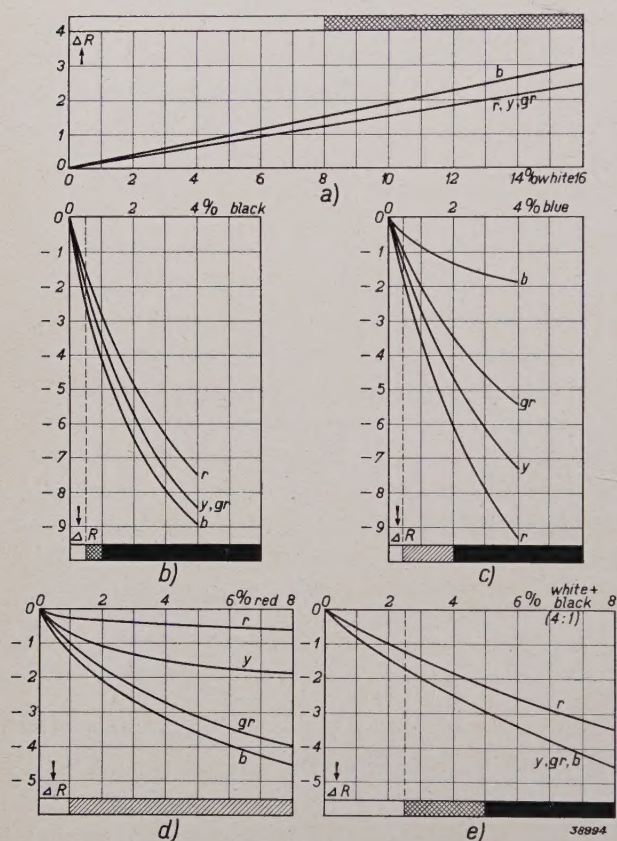


Fig. 7. In the original sample of a light grey lacquer the following reflection coefficients were found for the four wave lengths (in percent of the analogous coefficient for a magnesium oxide surface):  $r = 26.5\%$ ,  $g = 30.5\%$ ,  $gr = 32.5\%$ ,  $b = 36.1\%$ . The change  $\Delta R$  is plotted which is experienced by these values when one (or more) of the four component pigments of the lacquer is added in excess: a) addition of white, b) of black, c) of blue, d) of red, e) of white and black in the ratio 4 : 1. The shaded band with each figure indicates whether or not the colour and brightness impressions differ visibly from those for the original lacquer. At the percentages in the white part of the band colour and brightness are correct, in the simply shaded part only the colour differs, in the doubly shaded part, only the brightness, in the black part both factors differ.

At the same time during the measurements an estimation of colour impression and brightness of the lacquer surface was given. The question was answered (subjectively of course) of whether or not the colour and brightness deviate visibly from

those of the surface with the original lacquer. In figs. 7a-e this estimation is represented by the variously shaded band accompanying each figure: in the region of percentages where the band is left white it is impossible to observe either difference in colour or difference in brightness compared with the original lacquer. In the part shaded in one direction only, only the colour deviates; in the doubly shaded part only the brightness, finally in the black part both colour and brightness differ.

If, without bothering about the percentages of pigment, one considers more closely the relation between the measured values of  $\Delta R$  and the estimation of colour and brightness the following conclusions are reached. The colour impression is still "correct" (i.e. the same as that of the original lacquer) as long as the four values of  $\Delta R$  for  $r, g, gr$  and  $b$  all lie within an interval of 0.8. The brightness impression is still "correct" as long as at least one of the four values of  $\Delta R$  for  $r, g, gr$  or  $b$  lies below  $\Delta R = 1.2$ . If the subjective estimation given here is accepted by all concerned, the values may then be considered as the tolerances for the light grey lacquer in question<sup>4</sup>). If, for example, with a newly prepared portion of the mixture the relative reflection coefficients  $r = 25.5$  per cent,  $g = 30.5$  per cent,  $gr = 33.5$  per cent and  $b = 36.6$  per cent are measured, then the four values of  $\Delta R$  are  $-1.0$ ,  $0$ ,  $+1.0$  and  $+1.5$  respectively. The smallest of these is  $\Delta R = 0$  (namely for  $g$ ), the brightness impression will therefore be correct; the scattering of the values of  $\Delta R$  is, however, 2.5 (namely between  $-1.0$  for  $r$  and  $+1.5$  for  $b$ ), so that the colour deviation to be expected cannot be considered permissible.

In such a case it may also be deduced from the graphs how the error can be corrected. Further consideration, namely of the curves in fig. 7e, which were obtained by the addition of a mixture of white and black pigment, shows that the differences  $\Delta R$  can be calculated in good approximation simply by adding the values of  $\Delta R$  caused by each pigment separately. On the basis of the curves of figs. 7 a-d, therefore, it can be ascertained directly what reflection coefficients will be obtained upon addition of pigment to the lacquer which has been found unsuitable. In the case described, from the too large value of  $b$  and the too small value of  $r$

<sup>4</sup>) For lacquers of other colours quite different tolerance requirements may be reached, so that each case should be investigated separately.

<sup>5</sup>) In the mixing of pigments there is no question of an additivity for the different colours. This does not alter the fact, however, that with very small colour differences the mixing law here observed may hold for the reflection coefficients.



it could immediately be concluded that there was too little red pigment in the mixture. From fig. 7d it may now be read off that an addition of for instance 4 per cent of red pigment will give the values of  $\Delta R$  for  $r$ ,  $g$ ,  $gr$  and  $b$  of  $-0.4$ ,  $-1.5$ ,  $-2.7$  and  $-3.1$  respectively. This, added to the above-mentioned values of  $\Delta R$  gives the new values:  $-1.4$ ,  $-1.5$ ,  $-1.7$ ,  $-1.6$ . The new mixture satisfies the tolerance

requirement as far as the colour impression is concerned (the scattering is only 0.3), but not the requirement as to brightness. This can now still be corrected by adding about 10 per cent of white pigment which causes all four values of  $\Delta R$  to increase by about 1.7; the mixture obtained will have the values of  $\Delta R$ :  $+0.1$ ,  $0$ ,  $-0.2$ ,  $+0.3$ , and thereby satisfy both tolerance requirements.

## ABSTRACTS OF RECENT SCIENTIFIC PUBLICATIONS OF THE N.V. PHILIPS' GLOEILAMPENFABRIEKEN

An adequate number of reprints for the purpose of distribution is not available of those publications marked with an asterisk. Reprints of other publications may be obtained on applications to the Natuurkundig Laboratorium, N.V. Philips' Gloeilampenfabrieken, Eindhoven (Holland), Kastanjelaan.

- 1530:** C. J. Bakker: Fluctuations and electron inertia (*Physica* **8**, 23-43, Jan. 1941).

After having dealt with the spontaneous current fluctuations which occur in an electronic valve whose emission is limited by the space charge present in the neighbourhood of the cathode, the displacement currents are calculated which these fluctuations cause in the grids of the electronic valve (*cf.* also Philips techn. Rev. **6**, 129, 1941). These displacement currents are important in the use of electronic valves on short waves. For two different types of radio valves, namely a pentode and a four beam octode, the current fluctuations in the connection between one of the grids and the cathode are investigated experimentally as well as theoretically. The agreement between theory and experiment was found to be excellent.

- 1531:** M. J. O. Strutt and A. van der Ziel: The results of several electron inertia effects in electronic valves I. Theoretical explanations (*Physica* **8**, 81-108, Jan. 1941 (Original in German)).

The currents are studied which flow to the different electrodes of a vacuum tube when a small group of electrons passes from the cathode to the anode. This is further elaborated for the case where periodic current impulses leave the cathode. Under certain conditions the anode current will behave quite differently from these impulses. The same is true for a very high-frequency A.C. between control grid and cathode. This may lead to remarkable

phenomena such as frequency multiplication and rectification. For a harmonic alternating voltage on the control grid the values of input resistance and amplification factor are calculated. Simple formulae are derived for the displacement currents which occur due to electrons which pass close to the conductors in radio valves. A so-called centre of gravity hypothesis is hereby formulated which makes a simple calculation possible. Finally the possibility of transition time modulation is mentioned.

- 1532:** J. van Niekerk and M. S. C. Blik: Het prophylactische effect van éénmaal oraal of intramusculair, toegediende groote dosis bestraald provitamine D van dierlijke oorsprong bij rachitis. (The prophylactic effect of a single large dose of irradiated provitamine D of animal origin, administered orally or intermuscularly, in the case of rickets). *Ned. T. Geneesk.* **85**, 522-530, Feb. 1941).

Tests have been carried out for the purpose of studying the difference in action on groups of chicks of different amounts of irradiated provitamine D of animal origin administered in a single dose either through the beak or injected into a muscle. As a criterion of the activity of the vitamine D, its effectiveness against rachitic lesions in the growing skeleton has been chosen, as observed in X-ray photographs. The bone formation of different chicks is judged individually, and the protection



against rickets was determined separately as a number for each group of birds which received the same dose administered in the same way. The anti-rachitic action of the vitamine is found to depend upon the size of the single dose administered. Furthermore it was determined that an amount of vitamine D which is injected into a muscle acts more effectively and for a longer time than the same amount upon administration through the beak. Of a normal amount of vitamine D which during 12 successive weeks the chicks had consumed with their food, much too little was found to be retained to insure normal bone formation in the following weeks.

**1533:** J. M. Stevels: Physical properties of glasses I (Rec. Trav. chim. Pays Bas **60**, 85-86, Febr. 1941).

The way in which the specific volume of different kinds of glass depends upon the composition is discussed in this article.

**1534:** J. J. Went: Adsorption phenomena on massive metal surfaces measured by means of electrical contact resistances (Physica **8**, Febr. 1941).

For the contents of this article the reader may refer in general to: Philips techn. Rev. **5**, 238, 1940.

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